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BENDING OF THE PLATE WITH A CRACK IN PRESENCE OF PLASTIC ZONES AT ITS TOPS

Oksana Bilash

Hetman Petro Sahaidachnyi National Army Academy, Lviv, Ukraine

Summary. *A problem of biaxial bending under distributed bending moments at infinity of isotropic plate with the straight crack and the presence at its tops of plastic zones where Trasck's plasticity conditions are performed as a surface layer or plastic hinge, when the vectors of the external moments are perpendicular and parallel to the banks of the crack has been investigated. Using the methods of complex variable theory and the complex potentials of classic theory of plate bending, the solving of problem is reduced to the problem of linear coupling, their analytical solution has been received in the class of functions limited in the plastic zones tops.The length of plastic zone and divergence of crack banks at its top has been determined analytically and their numerical analysis for various parameters of the problem has been conducted.*

Key words: *plate, bending, crack, plastic zone, Trasck's plasticity condition (of plastic hinge).*

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Introduction. Plastic elements of structures are widely used in various industries, where with structural considerations they may have thin straight cuts that can be simulated by cracks and under the action of bending or any other load they are powerful stress concentrators, which reduces their durable characteristics.

It should be noted that the crack can be seen as a split, the banks of which during the bending load do not contact, this fact was taken into account by many researchers, as reflected in the monographs [1, 3 – 5, 7 – 9], while in [3, 4] the availability of plastic zones at its tops is taken into account, where the general statement of problems for the shell (plate) construction elements with such defects has been made under the influence of a specified load and the solution of problems is reduced to a system of singular integral equations.

In this paper, using the methods of theory of complex variable and complex potentials of the classical theory of plate bending, the problem about the biaxial bending of isotropic plate distributed by bending moments at infinity has been investigated, the vectors of which are parallel and perpendicular to the rectilinear crack at the tops of which plastic zones are available, where Trasck's plasticity conditions are performed in the form of a surface layer or the plastic hinge [3, 4]. The analytical solution of the problem in the class of functions bounded at the tops of plastic zones has been obtained and the corresponding dependences for the finding of length of plastic zone and differences of the banks of the crack at its top have been recorded, numerical analysis of the problem has been conducted.

The formulation of the problem. We will explore stress-strained state of isotropic plate $2h$ thick, the foundations of which are free from external load, with a through straight crack $2l$ length on its bilateral bending by its moments at infinity, vectors of which are parallel and perpendicular to the banks of the crack, and at its tops there are plastic zones, where Trasck's plasticity conditions are carried out in the form of a condition of plasticity of the surface layer or condition of plastic hinge and where constant bending moments operate M_0 [4]. The banks of the crack are free from external load and its dimensions are such that its banks during the bend are not in contact (see. Figure 1).

In the middle of the plate plane we choose a Cartesian coordinate system $Oxy\tilde{z}$ with the beginning in the center of the crack, by directing axis Ox on it, and the axis $O\tilde{z}$ perpendicular to this plane, where coordinate axis Oy is. The line of axis Ox, where a crack is placed, we will denote by L, plastic zones by – L_1, their length and ends by Δ, $-d$ and d in accordance.

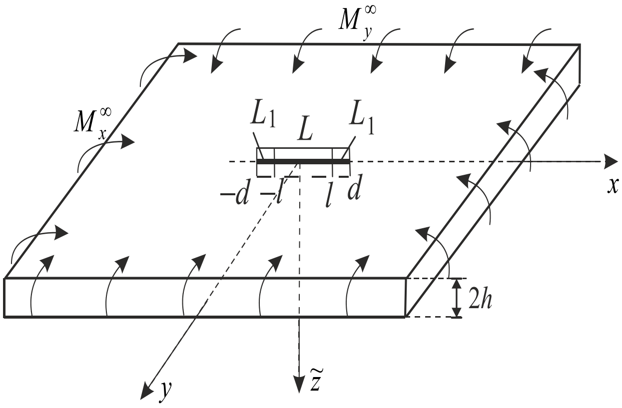

Figure 1. Chart of plate's load and placing of crack

According to the formulation of the problem we have the following boundary conditions

$$M_y^{\pm} = 0\,,\; N_y^{\pm} = 0\,,\; H_{xy}^{\pm} = 0\,,\; x \in L\,,$$

$$M_y^{\pm} = M_0\,,\; N_y^{\pm} = 0\,,\; H_{xy}^{\pm} = 0\,,\; x \in L_1\,, \tag{1}$$

where by the icons «+» and «–» threshold value of corresponding value is marked at $y \to \pm 0$, M_y – bending moment, N_y – cutting force, H_{xy} – torque.

Construction of the solution of the problem. When solving the problem we will use the classical theory of bending plates. We will introduce complex potentials $\Phi(z)$ and $\Omega(z)$ and having taken into account the dependencies [8]

$$\left(M_y + ic' + i\int_0^z P(\varepsilon)\partial\varepsilon\right) = m\left[\tilde{\kappa}\Phi(z) + \Omega(\bar{z}) - (z-\bar{z})\overline{\Phi'(z)}\right], \tag{2}$$

$$\partial_x(u + i\nu) = -\tilde{z}\left[\Phi(z) - \Omega(\tilde{z}) + (z-\tilde{z})\overline{\Phi'(z)}\right], \tag{3}$$

where $P(x) = N_y + \partial_x H_{xy}$ – is generalized in the sense of Kirchhoff cutting force; u and v – projections of vector of displacement on the axis Ox and Oy; c' – unknown constant; $m = -D(1-\upsilon)$, $\tilde{\kappa} = (3+\upsilon)/(1-\upsilon)$, $D = 2Eh^3/\left(3\left(1-\upsilon^2\right)\right)$; υ and E – respectively Poisson's ratio and Young's modulus of the material of the plate; $z = x + iy$, $i^2 = -1$, u and v – coordinates of the point; $\partial_x = \partial/\partial x$.

Introduced complex potentials at large $|z|$ can be represented as [8]

$$\Phi(z) = \tilde{\Gamma} + O\left(1/z^2\right),\; \Omega(z) = -\tilde{\Gamma} - \tilde{\Gamma}' + O\left(1/z^2\right), \tag{4}$$

where

$$\tilde{\Gamma} = -\frac{M_x^\infty + M_y^\infty}{4D(1+\upsilon)}, \ \tilde{\Gamma}' = \frac{M_x^\infty - M_y^\infty}{2m}, \tag{5}$$

M_x^∞ and $M_y^\infty -$ bending distributed moments at infinity.

On the basis of the boundary conditions (1) we can write

$$\left(M_y + i\int_0^x P(\varepsilon)\, d\varepsilon \right)^{\pm} = \begin{cases} 0, x \in L, \\ M_0/m, x \in L_1. \end{cases} \tag{6}$$

If we consider (2), then out of (6) we will get

$$\tilde{\kappa}\Phi^{\pm}(x) + \Omega^{\pm}(x) = \frac{ic'}{m} + \begin{cases} 0, x \in L, \\ M_0/m, x \in L_1. \end{cases} \tag{7}$$

From dependencies (7) we will obtain problems of linear conjugation in order to determine complex potentials $\Phi(z)$ and $\Omega(z)$

$$[\tilde{\kappa}\Phi(x) - \Omega(x)]^{+} - [\tilde{\kappa}\Phi(x) - \Omega(x)]^{-} = 0, \ x \in L + L_1,$$

$$\left(\tilde{\kappa}\Phi(x) + \Omega(x) - \frac{ic'}{m}\right)^{+} + \left(\tilde{\kappa}\Phi(x) + \Omega(x) - \frac{ic'}{m}\right)^{-} = \begin{cases} 0, x \in L, \\ \dfrac{2M_0}{m}, x \in L. \end{cases} \tag{8}$$

having solved them in the class of functions, limited at the tops of plastic zones [6], and taking into account the behavior of the functions $\Phi(z)$ and $\Omega(z)$ at the infinity (4), we will obtain

$$\Omega(z) = \tilde{\kappa}\Phi(z) - (\tilde{\kappa} + 1)\tilde{\Gamma} - \tilde{\Gamma}', \tag{9}$$

$$\tilde{\kappa}\Phi(z) + \Omega(z) - \frac{ic'}{m} = -\frac{M_0 X(z)}{m\pi} \int_{L_1} \frac{dx}{\sqrt{d^2 - x^2}\,(x - z)}, \tag{10}$$

where $X(z) = \sqrt{z^2 - d^2}$.

Taking into account the behavior of the potentials $\Phi(z)$ and $\Omega(z)$ at the infinity (4), from (10) we obtain

$$-(\tilde{\kappa} - 1)(M_y^\infty + M_x^\infty)/(4D(1+\upsilon)) + (M_y^\infty - M_x^\infty)/2m - ic'/m = \frac{M_0}{\pi m} \int_{L_1} \frac{dx}{\sqrt{d^2 - x^2}}. \tag{11}$$

By equating in (11) real and imaginary parts and by calculating the integral [2], we will obtain

$$c' = 0, \ d = l \sec(\pi M_y^\infty/(2M_0)). \tag{12}$$

In order to find M_0 we will use Trasck's plasticity condition in the form of a surface layer [3, 4] based on the lower plate

$$M_0 = \frac{2h^2}{3}\sigma_Y , \tag{13}$$

or the condition of a plastic hinge

$$M_0 = h^2\sigma_Y , \tag{14}$$

where σ_Y – is the border of fluidity of the material of the plate.

On the basis of the formulas (9) and (10) we will find

$$\Phi(z) = \frac{1}{2\tilde{\kappa}}\left[(\tilde{\kappa}+1)\tilde{\Gamma} + \tilde{\Gamma}' + \frac{M_0 X(z)}{\pi m i}\int_{L_1}\frac{dx}{X^+(x)(x-z)} \right]. \tag{15}$$

If we consider (9), then based on (3), we will have

$$\partial_x \left[(u+i\nu)^+ - (u+i\nu)^- \right]\Big|_{\tilde{z}=h} = -h(1+\tilde{\kappa})(\Phi^+(x) - \Phi^-(x)), \quad x \in L_1 . \tag{16}$$

By substituting (15) in (16) and taking into account [10], we will get

$$\partial_x (\nu^+ - \nu^-)\Big|_{\tilde{z}=h} = \frac{M_0 h(1+\tilde{\kappa})}{2\pi\tilde{\kappa}m}\cdot\left[\Gamma(d,x,l) - \Gamma(d,x,-l) \right], \quad x \in L_1 , \tag{17}$$

where

$$\Gamma(d,x,\xi) = \ln\frac{d^2 - x\xi - \sqrt{(d^2-x^2)(d^2-\xi^2)}}{d^2 - x\xi + \sqrt{(d^2-x^2)(d^2-\xi^2)}}. \tag{18}$$

Divergence of crack banks at its top we find by means of the formula

$$\delta(l) = \delta_\kappa = \int_d^l \partial_x \left(v^+ - v^- \right)\Big|_{\tilde{z}=h} dx. \tag{19}$$

After substituting (17) to (19) and calculating of the corresponding integral [10] we will have

$$\delta_\kappa = \frac{2\mathrm{M}_0 \left(1+\tilde{\kappa}\right) hl}{\pi\tilde{\kappa}m}\ln\frac{l}{d} . \tag{20}$$

Numerical analysis. Taking into account (12) – (14), (20), for the numerical analysis of the problem we will submit consolidated length of plastic zone $\varepsilon = \Delta/l$ and consolidated divergence of the banks of the crack on the lower base, at the top of the crack $\tilde{\delta} = (\delta_\kappa E)/(l\sigma_y)$, so using Trasck's plasticity condition as the condition of plasticity of the surface layer [4]

$$\varepsilon = \sec(\pi\tilde{\sigma})/2) - 1, \ \tilde{\delta} = 2\gamma , \tag{21}$$

or the condition of plastic hinge [4]

$$\varepsilon = \sec(\pi\tilde{\sigma})/3) - 1,\ \tilde{\delta} = 3\gamma\,, \tag{22}$$

where

$$\tilde{\sigma} = 3M_y^{\infty} \big/ (2h^2\sigma_Y)\,,\ \gamma = 4(1+\upsilon)\ln(1+\varepsilon)\big/(\pi(3+\upsilon)).$$

Numerical analysis of the problem has been performed, which is shown in fig. 2 and 3, where the solid lines on the basis of Trasck's plasticity condition in a form of a surface layer, and dashed lines – by using the condition of plastic hinge.

Fig. 2 shows graphical dependence of consolidated length of plastic zone ε on consolidated bending stress $\tilde{\sigma}$ at infinity. As can be seen from this figure at $\tilde{\sigma} < 0,5$ we observe a slight increase of consolidated length of plastic zone ε regardless of the conditions of plasticity. At the time when $\tilde{\sigma}$ is close to 1, the length of the consolidated plastic zone increases significantly, heading toward the infinity by using Trasck's plasticity condition as a surface layer, which we do not observe when using the condition of a plastic hinge, where the consolidated length ε is always finite.

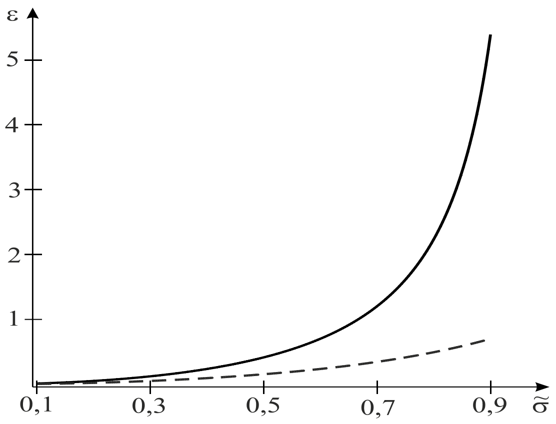

Figure 2. Graphic dependencies of the plastic zone length on the $\tilde{\sigma}$

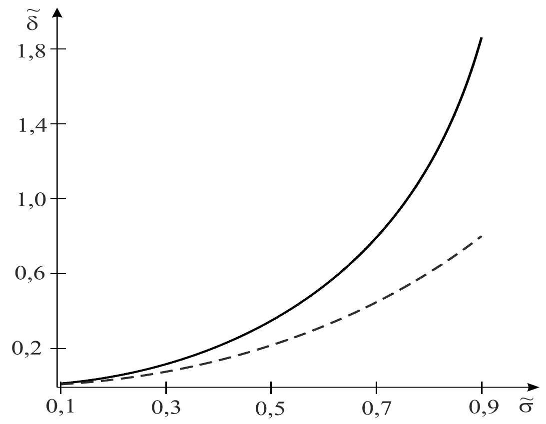

Figure 3. Graphic dependencies of the divergence of the crack banks on lower plate basis on $\tilde{\sigma}$

Fig. 3 shows a graphical dependence of consolidated divergence of the crack banks on the lower base in the top of the crack from $\tilde{\sigma}$. As we can see from this figure at small values of the external load M_y^{∞} divergence of the crack banks $\tilde{\delta}$ is insignificant, at growth M_y^{∞} $\tilde{\delta}$ grows, moreover, it is greater with the usage of Trasck's plasticity conditions in a form of a surface layer, than when using plastic hinge condition.

Conclusions. Numerical analysis has shown that the length of plastic zone and the divergence of crack banks on the bottom base plate is greater when using Trasck's plasticity condition in a form of a surface layer than when using the condition of plasticity in a form of plastic hinge with the same values $\tilde{\sigma}$. By increasing the value of the external load the length of plastic zone and the divergence of the crack banks increases. In addition, the bending moment at infinity M_x^{∞} has no effect neither on the length of plastic zone nor on the disclosure at the top of the crack when using such conditions of plasticity. It should be noted that based on δ_{κ} – the model [4, 7] the limit load can be determined at which the plate with the crack will be destroyed.

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УДК 539.3

ЗГИН ПЛАСТИНИ ЗІ ЩІЛИНОЮ ЗА НАЯВНОСТІ ПЛАСТИЧНИХ ЗОН У ЇЇ ВЕРШИНАХ

Оксана Білаш

Національна академія сухопутних військ імені гетьмана Петра Сагайдачного, Львів, Україна

Резюме. *Досліджено задачу про двовісний згин розподіленими згинальними моментами на нескінченності ізотропної пластини з прямолінійною щілиною за наявності у її вершинах пластичних зон, де виконуються умови пластичності Треска у вигляді поверхневого шару чи пластичного шарніра, коли вектори зовнішніх моментів паралельні й перпендикулярні до берегів щілини. Із використанням методів теорії функцій комплексної змінної та комплексних потенціалів класичної теорії згину пластин, розв’язок задачі зведено до задач лінійного спряження, отримано їх аналітичний розв’язок у класі функцій обмежених у вершинах пластичних зон. Визначено аналітично довжину пластичної зони та розходження берегів щілини у її вершині, проведено їх числовий аналіз за різних параметрів задачі.*

Ключові слова*: пластина, згин, щілина, пластичні зони, умова пластичності Треска (пластичного шарніра).*

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