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МЕХАНІКА ТА МАТЕРІАЛОЗНАВСТВО

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THE GENERALIZED MODEL OF INTERACTION OF ULTRASONIC SH-POLARIZATION WAVE WITH 2D STRAIN FIELD IN A PLANE LAYER

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Summary. *Within the framework of previously developed mathematical model of small elastic perturbations propagation in homogeneously strained elastic continuum a stationary problem for interaction of a narrow ultrasonic beam of SH polarization waves with an elastic layer under plane deformation has been formulated. In the problem the phenomenon of interference of reflected and refracted waves on surface of including small elastic perturbations in strained solid has been taken into account. The problem was solved by the iterative method. With use the approximation of geometric optics the parameters of reflected and refracted waves in the system «elastic half-space – the layer – elastic half-space» has been obtained. The influence of plane strain parameters on the coefficients of reflected and refracted waves has been studied.*

Key words: *heterogeneous deformed state, flat harmonic waves, boundary value problem, mean integral approximation, the parameters of reflected and refracted waves.*

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Problem setting. Ultrasound is used for diagnostics of stiffness and reliability of laminated structures under stresses that are stipulated both with external loadings and residual ones. The notion of acoustic elasticity is widely used here due to dependence of wave field parameters on distribution of deformation and stresses components along directions of propagation of wave bundles [1, 2]. In case the length of sounding wave is considerably shorter than normal length of stress field alteration, one can apply the approximation of geometric optics and the wave field can be regarded as a beam ensemble [1].

Within the elaborated earlier mathematical model the propagation of small elastic perturbations in heterogeneously strained elastic continuum there was formed a stationary problem of interaction between narrow ultrasonic beams ensemble of SH polarization waves with an elastic layer under flat deformation. The iteration method was used to solve this problem [3]. In the issue [4] due to such approach the medium integral deformations along the beam propagation as well as processes of reflection and refraction of waves with the layer were studied. The authors here came down to investigation of rather narrow wave bundle and large enough tilting angles while there is no interference of waves being reflected by upper and lower surfaces of the layer (Figure 1). The article [5] deals with comprehensive research of SH-wave angle where the interference is available.

Analysis of the latest researches and issues. The investigation of propagation, reflection and refraction of waves in heterogeneous laminated environment is dealt in a number of domestic and foreign issues [6 – 11], which describe the approaches to solving of certain

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problems of determining structural heterogeneities or initial stresses in solid bodies by means of data provided with ultrasonic measurements. Thus, in [6] they constructed a model to describe superficial acoustic waves of horizontal polarization in laminate anisotropic environment with its further application for non-fracturing diagnostics of laminates. The issue [7] investigates the reciprocal problem determining residual stresses on the database of acoustic tomography. In [8] with the purpose to study the practical possibilities of non-linear acoustic tomography they described the processes of waves' generation that were originated by 3rd degree non-linear interaction. The [9] is devoted to the problem of normal penetration flat sonic wave through flat composite layer consisting of a large number of elastic and viscous-elastic isotropic materials that go one after another. The [10] presents the new model of seismic wave propagation in viscous-elastic heterogeneous environment. In [11] the mathematical model of SH-wave propagation in laminates with mono-cell symmetry was derived. However, so far the problem of analytical correlations linking the parameters of reflected and refracted by objects under heterogeneous flat strained condition waves with deformation fields in it, and which are rather friendly to be used in ultrasonic tomography of stresses, has not been solved yet.

Research objectives. The article within the model of small elastic perturbations in heterogeneous strained elastic continuum [3] deals with the problem of interaction between wide ultrasonic beams ensemble of SH polarization waves with flat layer under condition of heterogeneous flat deformation. The authors investigated the case of such dip angle and bundle width when reflected by upper and lower layer surfaces waves coincide partially. On the basis of problem solution they carried out the quantitative analysis of reflection and refraction rates in the area of coincidence according to layer thickness for predetermined distributions of initial deformation components in the layer.

Task setting. There is studied an isotropic homogeneous elastic layers $\mathbf{S}: 0 < x_2 < h$, which splits two isotropic half-spaces $\mathbf{S}^{(1)}: -\infty < x_2 < 0$ and $\mathbf{S}^{(2)}: h < x_2 < \infty$ with weight densities $\rho^{(n)}$ and shift modules $\mu^{(n)}$ $(n = 1, 2)$ respectfully. The layer is in flat deformation condition and consequently becomes acoustically heterogeneous and anisotropic. Due to weight density ρ, Lame coefficients λ, μ and Murnaghan a, b, c the acoustic properties of the layer under its natural condition are determined with components of deformation tensor $e_{ij} = e_{ij}(x_1, x_2)$, $(i, j = 1, 2)$.

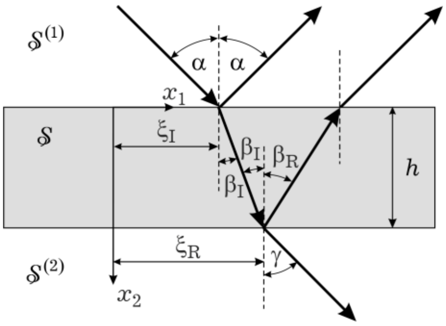

Figure 1.
Interaction of ultrasonic SH-polarization wave with 2D strain field in a plane layer

Suppose in half-space $\mathbf{S}^{(1)}$ along the direction creating the angle α with axis $0x_2$, ultrasonic bundle of SH-polarization waves propagates and perturbs acoustic motion in some space area $\mathbf{V}^{(1)} \subset \mathbf{S}^{(1)}$ with cross-cut dimension d, which is much more large than wave length $\lambda^{(1)} = 2\pi C^{(1)} / \omega$, where $C^{(1)} = \sqrt{\mu^{(1)} / \rho^{(1)}}$, ω – frequency. Under such conditions the acoustic motion process in the area $\mathbf{V}^{(1)}$ can be described by flat wave with quite high accuracy:

$$w_{\mathrm{I}}^{(1)}\left(x_1,x_2,t\right)=\dot{W}_{\mathrm{I}}^{(1)}\exp\left(\mathrm{i}\,K^{(1)}\left(x_1\sin\alpha+x_2\cos\alpha\right)\right)\exp\left(-\mathrm{i}\,\omega t\right). \quad (1)$$

Here $\dot{W}_{\mathrm{I}}^{(1)}$ – complex constant, $K^{(1)}=\omega/C^{(1)}$ – wave number.

Waves reflected and refracted by surfaces $x_2=0$ та $x_2=h$ will be also harmonic in time and retain SH-polarization. The waves reflected into half-space $\mathbf{S}^{(1)}$ and refracted into $\mathbf{S}^{(2)}$ will be flat ones

$$w_{\mathrm{R}}^{(1)}\left(x_1,x_2,t\right)=\dot{W}_{\mathrm{R}}^{(1)}\exp\left(\mathrm{i}\,K^{(1)}\left(x_1\sin\alpha-x_2\cos\alpha\right)\right)\exp\left(-\mathrm{i}\,\omega t\right),$$

$$w_{\mathrm{I}}^{(2)}\left(x_1,x_2,t\right)=\dot{W}_{\mathrm{I}}^{(2)}\exp\left(\mathrm{i}\,K^{(2)}\left(x_1\sin\gamma+x_2\cos\gamma\right)\right)\exp\left(-\mathrm{i}\,\omega t\right), \quad (2)$$

where $\dot{W}_{\mathrm{R}}^{(1)}$, $\dot{W}_{\mathrm{I}}^{(2)}$ – complex constants, $K^{(2)}=\omega/C^{(2)}$, $C^{(2)}=\sqrt{\mu^{(2)}/\rho^{(2)}}$.

The wave $w_{\mathrm{I}}\left(x_1,x_2,t\right)$, which was retracted into the layer by the surface $x_2=0$, and $w_{\mathrm{R}}\left(x_1,x_2,t\right)$, reflected into it by the surface $x_2=h$, will be represented as

$$w_{\chi}\left(x_1,x_2,t\right)=\dot{W}_{\chi}(x_1,x_2)\exp\left(-\mathrm{i}\,\omega t\right),\quad (\chi=\mathrm{I},\mathrm{R}). \quad (3)$$

Having taken into consideration that caused by deformation acoustic anisotropy and heterogeneity of the layer are small, we accept the angle dispersion of the wave bundle in acoustically heterogeneous area $\mathbf{S}$ to be inconsiderable.

As they use small amplitude waves in acoustic control, the amplitudes $\dot{W}_{\chi}(x_1,x_2)$ of waves $w_{\chi}\left(x_1,x_2,t\right)$ ($\chi=\mathrm{I},\mathrm{R}$) are also small and for their recording one can use the elaborated by the authors model of small elastic perturbation propagation in heterogeneously strained continuum [3]. Within this model the equation to determine the amplitudes of reflected and retracted SH-waves will be as follows:

$$\begin{gathered}\partial\left(\left[\mu+2be_1+ce_{11}\right]\partial/\partial x_1+ce_{12}\,\partial/\partial x_2\right)\dot{W}_{\chi}/\partial x_1+\\+\left[\partial\left(\left[\mu+2be_1+ce_{22}\right]\partial/\partial x_2+ce_{12}\,\partial/\partial x_1\right)/\partial x_2+\rho\omega^2\right]\dot{W}_{\chi}=0\end{gathered}, \quad (4)$$

where $e_1=e_{11}+e_{22}$.

On the velocity suppression $x_2=0$ та $x_2=h$ the condition of ideal mechanic contact for elastic perturbations are met.

Their result is correlations linking the waves' amplitudes:

$$\left(\dot{W}_{\mathrm{I}}^{(1)}+\dot{W}_{\mathrm{R}}^{(1)}\right)\exp\left(\mathrm{i}\,K^{(1)}x_1\sin\alpha\right)=\dot{W}_{\mathrm{I}}\left(x_1,0\right)+\dot{W}_{\mathrm{R}}\left(x_1,0\right),$$

$$\mu^{(1)}\,\mathrm{i}\,K^{(1)}\cos\alpha\left(\dot{W}_{\mathrm{I}}^{(1)}-\dot{W}_{\mathrm{R}}^{(1)}\right)\exp\left(\mathrm{i}\,K^{(1)}x_1\sin\alpha\right)=s_{\mathrm{I}}\left(x_1,0\right)+s_{\mathrm{R}}\left(x_1,0\right); \quad (5)$$

$$\dot{W}_{\mathrm{I}}\left(x_1,h\right)+\dot{W}_{\mathrm{R}}\left(x_1,h\right)=\dot{W}_{\mathrm{I}}^{(2)}\exp\left(\mathrm{i}\,K^{(2)}\left(x_1\sin\gamma+h\cos\gamma\right)\right),$$

$$s_{\mathrm{I}}\left(x_1,h\right)+s_{\mathrm{R}}\left(x_1,h\right)=\mathrm{i}\,\mu^{(2)}K^{(2)}\cos\gamma\dot{W}_{\mathrm{I}}^{(2)}\exp\left(\mathrm{i}\,K^{(2)}\left(x_1\sin\gamma+h\cos\gamma\right)\right), \tag{6}$$

where $s_{\chi}\left(x_1,x_2\right)=\left(\left(\mu+2be_1+ce_{22}\right)\partial/\partial x_2+ce_{12}\,\partial/\partial x_1\right)\dot{W}_{\chi}\left(x_1,x_2\right)$.

Equation (4), alongside with conditions (5), (6) determine the acoustic motion in the layer $\mathbf{S}$ in presentation (3).

From obtained mathematical model (4), (5) one can derive the problem solved in [4], having put $\dot{W}_{\mathrm{R}}\left(x_1,0\right)=0$ $s_{\mathrm{R}}\left(x_1,0\right)=0$ in marginal conditions (5). The case of normal dipping of sounding SH-wave [5] we will get from supposition in equations (4), (5) $\alpha=0$.

2. Iterative method of problem solution. To solve the formulated problem we will use the iteration scheme suggested in [3]. With this purpose we will rewrite the equation for the wave $w_{\mathrm{I}}\left(x_1,x_2,t\right)$ according to local Cartesian display $\mathbf{K}_{\mathrm{I}}=\left\{x,y,x_3\right\}$, obtained from original coordinates $\mathbf{K}=\left\{x_1,x_2,x_3\right\}$ with the turn in the plane x_1Ox_2 on the angle β_{I} and translation of computing point on the vector $\left(\xi_{\mathrm{I}},0\right)$. The wave $w_{\mathrm{R}}\left(x_1,x_2,t\right)$ will be studied according to frame of axis $\mathbf{K}_{\mathrm{R}}=\left\{x,y,x_3\right\}$, obtained from initial system $\mathbf{K}$ by the turning in plane x_1Ox_2 on the angles $\left(\pi-\beta_{\mathrm{R}}\right)$ and translation of computing origin on the vector $\left(\xi_{\mathrm{R}},h\right)$ respectfully. Here $\xi_{\mathrm{R}}=\xi_{\mathrm{I}}+h\,\mathrm{tg}\,\beta_{\mathrm{I}}$. The methods to determine the angles β_{I} and β_{R} were described earlier [12].

The equations in new frame of axis for refracted ($\chi=\mathrm{I}$) and reflected ($\chi=\mathrm{R}$) waves will be as follows

$$\left[\partial\left(\left\{\mu+\left(2b+c\right)\varepsilon_{11}+2b\varepsilon_{22}\right\}\partial/\partial x\right)\Big/\partial x+\hat{N}+\rho\omega^2\right]\dot{W}_{\chi}\left(x,y\right)=0\,, \tag{7}$$

where $\varepsilon=\varepsilon\left(x,y\right)=\varepsilon_{11}+\varepsilon_{22}$; $\hat{N}$ – differential operator

$$\hat{N}=c\left(\partial\left[\varepsilon_{12}\,\partial/\partial y\right]/\partial x+\partial\left[\varepsilon_{12}\,\partial/\partial x\right]/\partial y\right)+\partial\left[\left(2b\varepsilon+c\varepsilon_{22}\right)\partial/\partial y\right]/\partial y\,;$$

$$\begin{pmatrix}\varepsilon_{11}\left(x,y\right)\\ \varepsilon_{22}\left(x,y\right)\\ \varepsilon_{12}\left(x,y\right)\end{pmatrix}=\begin{pmatrix}e_{11}\sin^2\beta_{\mathrm{I}}+e_{22}\cos^2\beta_{\mathrm{I}}+e_{12}\sin2\beta_{\mathrm{I}}\\ e_{11}\cos^2\beta_{\mathrm{I}}+e_{22}\sin^2\beta_{\mathrm{I}}-e_{12}\sin2\beta_{\mathrm{I}}\\ 0.5\left(e_{22}-e_{11}\right)\sin2\beta_{\mathrm{I}}-e_{12}\cos2\beta_{\mathrm{I}}\end{pmatrix}\ \text{for}\ \chi=\mathrm{I},$$

$$\begin{pmatrix}\varepsilon_{11}\left(x,y\right)\\ \varepsilon_{22}\left(x,y\right)\\ \varepsilon_{12}\left(x,y\right)\end{pmatrix}=\begin{pmatrix}e_{11}\sin^2\beta_{\mathrm{R}}+e_{22}\cos^2\beta_{\mathrm{R}}-e_{12}\sin2\beta_{\mathrm{R}}\\ e_{11}\cos^2\beta_{\mathrm{R}}+e_{22}\sin^2\beta_{\mathrm{R}}+e_{12}\sin2\beta_{\mathrm{R}}\\ 0.5\left(e_{11}-e_{22}\right)\sin2\beta_{\mathrm{R}}-e_{12}\cos2\beta_{\mathrm{R}}\end{pmatrix}\ \text{for}\ \chi=\mathrm{R}. \tag{8}$$

Functions $\varepsilon_{ij}:\left(i,j=1,2\right)$ determining the distribution of deformation components in the layer will be presented as $\varepsilon_{ij}\left(x,y\right)=\bar{\varepsilon}_{ij}+\tilde{\varepsilon}_{ij}\left(x,y\right)$, where $\bar{\varepsilon}_{ij}$ – average integral values of deformation components in directions $\boldsymbol{n}_{\mathrm{I}}=\{\sin\beta_{\mathrm{I}},\cos\beta_{\mathrm{I}},0\}$ (for $\chi=\mathrm{I}$) and $\boldsymbol{n}_{\mathrm{R}}=\{\sin\beta_{\mathrm{R}},-\cos\beta_{\mathrm{R}},0\}$ (for $\chi=\mathrm{R}$):

$$\bar{\varepsilon}_{ij}\left(\beta_{\chi}\right)=\frac{\cos\beta_{\chi}}{h}\int\limits_0^{h/\cos\beta_{\chi}}\varepsilon_{ij}\left(s,0\right)ds\,,\ \left(\chi=\mathrm{I,R}\right). \tag{9}$$

Having applied iteration scheme for the equation (7), we obtain the series of equations with constant coefficients:

$$\left(\left[\mu+(2b+c)\bar{\varepsilon}-c\bar{\varepsilon}_{22}\right]\partial^2/\partial x^2+\rho\omega^2\right)\dot{W}_\chi^{[p+1]}=-\left(\hat{T}+\hat{N}\right)\dot{W}_\chi^{[p]}, \quad (10)$$

where $\hat{T}=\partial\left[\left((2b+c)\tilde{\varepsilon}_{11}(x,y)+2b\tilde{\varepsilon}_{22}(x,y)\right)\partial/\partial x\right]/\partial x$,
$p=0,1,2,\ldots$ – iteration number.

The problem solution will be selected as zero approximation

$$\left(\mu+(2b+c)\bar{\varepsilon}-c\bar{\varepsilon}_{22}\right)\partial^2\dot{W}_\chi/\partial x^2+\rho\omega^2\dot{W}_\chi=0. \quad (11)$$

The upper index $\left(\mu+(2b+c)\bar{\varepsilon}-c\bar{\varepsilon}_{22}\right)\partial^2\dot{W}_\chi/\partial x^2+\rho\omega^2\dot{W}_\chi=0$ will be here and further omitted for zero iteration.

Solutions for the equations (11) are as follows

$$\dot{W}_\chi(x)=\dot{W}_\chi\exp\left(\mathrm{i}\,K_\chi x\right), \quad (12)$$

де $$K_\chi=\omega/\bar{C}_\chi,\quad \bar{C}_\chi=\sqrt{\left[\mu+(2b+c)\bar{\varepsilon}(\beta_\chi)-c\bar{\varepsilon}_{22}(\beta_\chi)\right]/\rho}. \quad (13)$$

For solutions (12) the correlations (5), (6) will be presented as a system of equations for unknown amplitudes $\dot{W}_\mathrm{R}^{(1)}$, $\dot{W}_\mathrm{I}$, $\dot{W}_\mathrm{R}$, $\dot{W}_\mathrm{I}^{(2)}$:

$$\dot{W}_\mathrm{I}^{(1)}+\dot{W}_\mathrm{R}^{(1)}=\dot{W}_\mathrm{I}+\dot{W}_\mathrm{R},\quad K^{(1)}\mu^{(1)}\left(\dot{W}_\mathrm{I}^{(1)}-\dot{W}_\mathrm{R}^{(1)}\right)\cos\alpha=\dot{W}_\mathrm{I}K_\mathrm{I}\sigma_\mathrm{I}+\dot{W}_\mathrm{R}K_\mathrm{R}\sigma_\mathrm{R},$$

$$\dot{W}_\mathrm{I}v_\mathrm{I}+\dot{W}_\mathrm{R}v_\mathrm{R}=\dot{W}_\mathrm{I}^{(2)}v_\mathrm{I}^{(2)};\qquad \dot{W}_\mathrm{I}v_\mathrm{I}\sigma_\mathrm{I}K_\mathrm{I}+\dot{W}_\mathrm{R}v_\mathrm{R}\sigma_\mathrm{R}K_\mathrm{R}=\dot{W}_\mathrm{I}^{(2)}\mu^{(2)}v_\mathrm{I}^{(2)}K^{(2)}\cos\gamma. \quad (14)$$

Here are used the indicators:

$$\sigma_\mathrm{I}=\rho\bar{C}_\mathrm{I}^{\,2}\cos\beta_\mathrm{I}+c\bar{\varepsilon}_{12}(\beta_\mathrm{I})\sin\beta_\mathrm{I};\quad \sigma_\mathrm{R}=c\bar{\varepsilon}_{12}(\beta_\mathrm{R})\sin\beta_\mathrm{R}-\rho\bar{C}_\mathrm{R}^{\,2}\cos\beta_\mathrm{R};$$

$$v_\mathrm{I}=\exp\left(\mathrm{i}\,K_\mathrm{I}h\cos\beta_\mathrm{I}\right),\quad v_\mathrm{R}=\exp\left(-\mathrm{i}\,K_\mathrm{R}h\cos\beta_\mathrm{R}\right),\quad v_\mathrm{I}^{(2)}=\exp\left(\mathrm{i}\,K^{(2)}h\cos\gamma\right).$$

In general case the equations system (14) has the following solutions:

$$\frac{\dot{W}_\mathrm{I}}{\dot{W}_\mathrm{I}^{(1)}}=\frac{2Z^{(1)}v_\mathrm{R}\left(Z_\mathrm{R}+Z^{(2)}\right)}{ZN};\qquad \frac{\dot{W}_\mathrm{R}}{\dot{W}_\mathrm{I}^{(1)}}=\frac{-2Z^{(1)}v_\mathrm{I}\left(Z^{(2)}-Z_\mathrm{I}\right)}{ZN};$$

$$\frac{\dot{W}_\mathrm{R}^{(1)}}{\dot{W}_\mathrm{I}^{(1)}}=\frac{v_\mathrm{R}\left(Z_\mathrm{R}+Z^{(2)}\right)\left(Z^{(1)}-Z_\mathrm{I}\right)-v_\mathrm{I}\left(Z^{(2)}-Z_\mathrm{I}\right)\left(Z^{(1)}+Z_\mathrm{R}\right)}{ZN};$$

$$\frac{\dot{W}_\mathrm{I}^{(2)}}{\dot{W}_\mathrm{I}^{(1)}}=\frac{2Z^{(1)}v_\mathrm{I}v_\mathrm{R}\left(Z_\mathrm{R}+Z_\mathrm{I}\right)}{v_\mathrm{I}^{(2)}ZN}, \quad (15)$$

where $Z^{(1)} = \rho^{(1)} C^{(1)} \cos\alpha$, $Z_{\mathrm{I}} = \rho \bar{C}_{\mathrm{I}} \cos\beta_{\mathrm{I}} + c\bar{\varepsilon}_{12}(\beta_{\mathrm{I}}) \sin\alpha / C^{(1)}$,

$$Z_{\mathrm{R}} = \rho \bar{C}_{\mathrm{R}} \cos\beta_{\mathrm{R}} - c\bar{\varepsilon}_{12}(\beta_{\mathrm{R}}) \sin\alpha / C^{(1)}, \quad Z^{(2)} = \rho^{(2)} C^{(2)} \cos\gamma,$$

$$ZN = v_{\mathrm{R}} \left(Z^{(1)} + Z_{\mathrm{I}} \right) \left(Z_{\mathrm{R}} + Z^{(2)} \right) - v_{\mathrm{I}} \left(Z^{(1)} - Z_{\mathrm{R}} \right) \left(Z^{(2)} - Z_{\mathrm{I}} \right).$$

Two last correlations (15) determine the refraction coefficients and propagation of flat SH-wave through the layer with initial deformations. Under condition that the layer in unstressed and half-spaces it splits have the same acoustic properties, the expressions (15) coincide with known ones [13].

Research results. The obtained in zero approximation solution was used for investigation of reflection and refraction of waves in the system consisting of two isotropic half-spaces divided with a layer with residual stresses. They studied the residual stresses stipulated with transposition shift, which is active within the section $x_1 = 0$. During growth of distance from the boundary $x_1 = 0$ the stresses decrease fast. Solution of this problem by means of method of homogenous solutions was presented in [3].

The elastic properties of materials were selected in order to avoid critical angles during reflection/refraction on the boundaries of velocity suppression. The materials of half-space $\mathbf{S}^{(1)}$ and layer $\mathbf{S}$ were considered to be identical with $\rho^{(1)} = \rho = 2.77 \cdot 10^3, \left[кг/м^3 \right]$, $\mu^{(1)} = \mu = 2.74 \cdot 10^{10}, \left[H/м^2 \right]$, $b = -4.802 \cdot 10^{10}, \left[H/м^2 \right]$, $c = -3.361 \cdot 10^{10}, \left[H/м^2 \right]$, and for $\mathbf{S}^{(2)}$ it was accepted that $\rho^{(2)} = 8.9 \cdot 10^3, \left[кг/м^3 \right]$, $\mu^{(2)} = 4.506 \cdot 10^{10}, \left[H/м^2 \right]$.

The influence of residual stresses upon parameters of wave field was determined via the wave reflection coefficient R_a by the surface $x_2 = 0$ into half-space $\mathbf{S}^{(1)}$ (Figure.1) and refraction coefficientта I_a by the surface $x_2 = h$ into half-space $\mathbf{S}^{(2)}$. Figures 2 and 3 display the dependencies of surpluses δR_a and δI_a of these coefficients stipulated by available residual stresses from the coordinate x_1 of the point where the beam enters the layer for different values of waves diapason. These surpluses were determined as

$$dR_a = \frac{\left| \dot{W}_{\mathrm{R}}^{(1)} \right| - \left| \dot{W}_{0\mathrm{R}}^{(1)} \right|}{\left| \dot{W}_{\mathrm{I}}^{(1)} \right|}, \qquad dI_a = \frac{\left| \dot{W}_{\mathrm{I}}^{(2)} \right| - \left| \dot{W}_{0\mathrm{I}}^{(2)} \right|}{\left| \dot{W}_{\mathrm{I}}^{(1)} \right|}$$

where $\dot{W}_{0\mathrm{R}}^{(1)}$, $\dot{W}_{0\mathrm{I}}^{(2)}$ – complex amplitudes of reflected from unstressed layer and refracted by it waves.

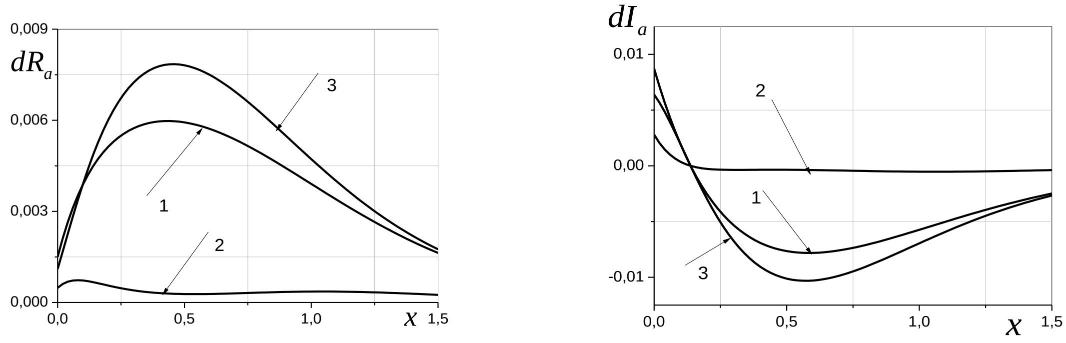

Figure 2. Dependencies of reflection coefficient δR_a and refraction coefficient δI_a surpluses on the coordinate of beam entering point in a layer at $h/\lambda = 0.352, 1.007, 1.252$

The calculations were carried out for the fixed dip angle $\alpha = \pi/8$. Numbers 1, 2, 3 mark the graphs for different layer thicknesses $h/\lambda = 0.352, 1.007, 1.252$ (Figure 2) and $h/\lambda = 2.266, 10.07, 12.335$ (Figure 3).

If the layer thickness equals approximately the number of half-waves (Figure 2), the availability of initial deformations in the layer does not impact the coefficients of reflection and refraction. In other cases (graph 1, Figure 3) we see the influence on the parameters of reflected/refracted waves of initial strained condition of the layer.

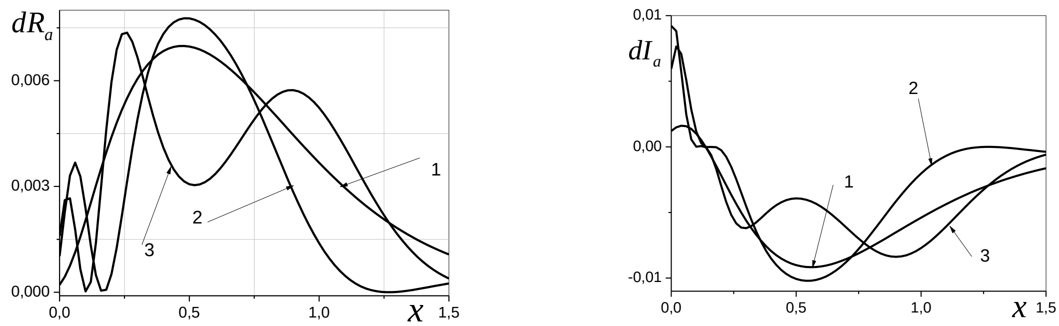

Figure 3. Dependencies of reflection coefficient δR_a and refraction coefficient δI_a surpluses on the coordinate of beam entering point in a layer at $h/\lambda = 2.266, 10.07, 12.335$

Conclusions. The problem for interaction between ultrasonic wave bundle of SH-polarization and an elastic layer under heterogeneous flat deformation has been studied. The research was carried out within the framework of iterative method taking into consideration the phenomenon of interference between wave bundles that were reflected by upper and lower surfaces of the layer. The authors investigated the case of residual stresses that appear at pre-determined on a plane $x_1 = 0$ displacement shifts being localized around this plane and decreasing fast at increasing of distance from it. The numerous experiments indicated that parameters of the wave field depend greatly from residual stresses and frequency of sounding waves.

The obtained results can be used for development of nondestructive ultrasonic methods of determination of residual stresses in construction units of corresponding configuration.

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УЗАГАЛЬНЕНА МОДЕЛЬ ВЗАЄМОДІЇ SH-ХВИЛІ ІЗ НАПРУЖЕНИМ ПЛОСКИМ ШАРОМ

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Резюме. *У межах теорії поширення малих пружних збурень у неоднорідно деформованому пружному континуумі розв'язану раніше задачу про взаємодію гармонічної плоскої SH-хвилі із пружним шаром, що перебуває у стані плоскої деформації, сформульовано із урахуванням інтерференції на межі входження збурення у деформоване середовище. У середньоінтегральному вздовж напрямку поширення збурення наближенні поля неоднорідних початкових деформацій отримано вирази для коефіцієнтів відбивання та заломлення за амплітудою від неоднорідно деформованого пружного шару.*

Ключові слова: *неоднорідний деформований стан, плоскі гармонічні хвилі, крайова задача, середньоінтегральні наближення поля деформацій, відбивання/заломлення хвиль.*

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