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ROBUST STABILITY AND EVALUATION OF THE QUALITY FUNCTIONAL OF LINEAR DISCRETE SYSTEMS WITH MATRIX UNCERTAINTY

Andrii Aliluiko; Valerii Yeromenko

Ternopil National Economic University, Ternopil, Ukraine

Summary. *New methods for analysis of robust stability and optimization of discrete output feedback control systems are developed. Sufficient stability conditions of the zero state are formulated with the joint quadratic Lyapunov function for control systems with uncertain coefficient matrices and a measured output feedback. The solution of a problem of robust stabilization and evaluation of the quadratic performance criterion for linear discrete systems with matrix uncertainty are proposed. The example of a stabilization two-masse mechanical system is showed.*

Keywords*: robust stability, matrix uncertainty, discrete systems, Lyapunov function, output feedback.*

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Problem setting. In applied problems of analysis and synthesis of real objects, one often uses systems of differential and difference equations with uncertain components (parameters, functions and random perturbation) (see, e.g., [1] – [6]). This focuses on the analysis and achievement of performance index of such systems particularly robust stability and optimality.

As set robust stability of dynamic systems we mean parametric or functional set characterizing uncertainty of the given structure of the system and its control components. In particular, in the uncertain linear models matrices of coefficients and feedback may belong to some given sets in the corresponding spaces (intervals, polytopes, affine and ellipsoidal families of matrices, etc.).

The problem of robust stabilization of the control system is to build a static or dynamic control to ensure the asymptotic stability for equilibrium states of the closed-loop system with arbitrary values of uncertain components.

Analysis of recent research and publications. Numerous works the problem of robust stabilization of control system is reduced to solving systems of linear matrix inequalities. In the works [3], [7], [8] find sufficient stability conditions for linear controllable systems with uncertain matrices of coefficients and feedback with respect to measurable output in terms of linear matrix inequalities. A survey of problems and known methods of robust stability analysis and stabilization of feedback control systems can be found in [9] – [11].

The aim of the research is to develop new methods of robust stability analysis and robust stabilization of linear difference systems with limited at a norm of matrix uncertainties and static measurable output feedback.

Robust stabilization of nonlinear control systems. Consider a linear dynamical control system with discrete time which describing difference equations in the form:

$$x_{t+1} = (A + \Delta A_t)x_t + (B + \Delta B_t)u_t, \quad y_t = Cx_t + Du_t, \tag{1}$$

where $x_t \in \mathrm{R}^n$, $u_t \in \mathrm{R}^m$ and $y_t \in \mathrm{R}^l$ are state, control, and observable object output vectors respectively, $t = 0,1,2,\ldots$, A, B, C and D are constant matrices of corresponding sizes $n \times n$, $n \times m$, $l \times n$ i $l \times m$, and

$$\Delta A_t = F_A \Delta_{At} H_A, \ \ \Delta B_t = F_B \Delta_{Bt} H_B,$$

where F_A, F_B, H_A, H_B are constant matrices of corresponding sizes and matrices uncertainties Δ_{At} and Δ_{Bt} satisfy the constraints

$$\|\Delta_{At}\| \le 1, \ \|\Delta_{Bt}\| \le 1 \text{ or } \|\Delta_{At}\|_F \le 1, \ \|\Delta_{Bt}\|_F \le 1, \quad t = 0,1,2,\ldots .$$

Hereinafter, $\|\cdot\|$ is Euclidean vector norm and spectral matrix norm, $\|\cdot\|_F$ is matrix Frobenius norm, I_n is the unit $n \times n$ matrix, $X = X^T > 0$ (≥ 0) is a positive (nonnegative) definite symmetric matrix. To simplify the records of the matrices dependency on t we will omit. For matrices B and C, that have full rank with respect to columns and rows respectively. We control the system (1) with output feedback:

$$u_t = K y_t, \ \ K = K_0 + \tilde{K}, \ \ \tilde{K} \in \mathrm{E}, \tag{2}$$

where E is an ellipsoidal set of matrices in the space $\mathrm{R}^{m \times l}$

$$\mathrm{E} = \{K : K^T P K \le Q\}, \tag{3}$$

where $P = P^T > 0$ and $Q = Q^T > 0$ are symmetric positive definite matrices of corresponding sizes $m \times m$ and $l \times l$.

According to (1) – (3), the following inequality must hold:

$$\left[x_t^T, \, u_t^T\right] \begin{bmatrix} C^T Q C - C^T K_0^T P K_0 C & C^T Q D + C^T K_0^T P G \\ D^T Q C + G^T P K_0 C & \Delta \end{bmatrix} \begin{bmatrix} x_t \\ u_t \end{bmatrix} \ge 0 ,$$

where $\Delta = D^T Q D - G^T P G$, $G = I_m - K_0 D$. We assume that

$$\Delta < 0 . \tag{4}$$

Then $x_t = 0$ implies $u_t = 0$, and $x_t \equiv 0$ is an equilibrium state for the system.

The problem is to construct conditions under which the zero state of the closed-loop control system (1) and (2) is Lyapunov asymptotically stable for every matrix $\tilde{K} \in \mathrm{E}$. Matrix K_0 is chosen for the purposes of stabilization, e.g., in case when the zero state of the system (1) without control ($u_t = 0$) is unstable.

$$x_{t+1} = M_0 x_t, \ \ M_0 = A + \Delta A + (B + \Delta B)(I_m - K_0 D)^{-1} K_0 C . \tag{5}$$

Matrix K_0 can be obtained with methods described in [12].

We introduce on the set of matrices $\mathsf{K} = \{K : \det(I_m - KD) \neq 0\}$ a nonlinear operator

$$\mathsf{D} : \mathsf{R}^{m \times l} \to \mathsf{R}^{m \times l}, \quad \mathsf{D}(K) = (I_m - KD)^{-1} K \equiv K(I_l - DK)^{-1}.$$

For the operator D the property is performed [12]: if $K_1 \in \mathsf{K}$, $K_2 \in \mathsf{K}$ and $K_3 = (I_m - K_1 D)^{-1} K_2 \in \mathsf{K}$ then

$$K_1 + K_2 \in \mathsf{K} \text{ and } \mathsf{D}(K_1 + K_2) \equiv \mathsf{D}(K_1) + \mathsf{D}(K_3)\left[I_l + D\mathsf{D}(K_1)\right]. \tag{6}$$

Under assumption (4) matrix G must be nondegenerate. Therefore values of the operator $\mathsf{D}(K_0) = (I_m - K_0 D)^{-1} K_0$ are defined. If $\tilde{K} \in \mathsf{E}$ then values of $\mathsf{D}(K)$ and $\mathsf{D}(\hat{K})$ are also defined, where $\hat{K} = G^{-1}\tilde{K}$. Indeed, under conditions (2) and (4) we have

$$D^T \tilde{K}^T P \tilde{K} D \leq D^T Q D < G^T P G, \quad F^T P F < P^{-1},$$

where $F = \tilde{K} D G^{-1}$ and $P > 0$. Therefore $\rho(F) < 1$, and matrix $I_m - F$ is nondegenerate, and hence matrices $I_m - KD = (I_m - F)G$ and $I_m - \hat{K}D = G^{-1}(I_m - KD)$ are nondegenerate as well.

Thus we exclude a control vector from relations (1) and (2) with restriction (4) and we get system

$$x_{t+1} = M x_t, \quad M = A + \Delta A + (B + \Delta B)\mathsf{D}(K)C. \tag{7}$$

Separately the zero equilibrium state of system (5) for $K = K_0$ should be asymptotically stable.

Using following statements, we will receive a solution of the formulated problem by means of methods of quadratic Lyapunov function.

Lemma 1. [12] *Suppose that the following matrix inequalities hold:*

$$\begin{bmatrix} R - P & D^T \\ D & -Q^{-1} \end{bmatrix} < 0, \quad \begin{bmatrix} W & U^T & V^T \\ U & R - P & D^T \\ V & D & -Q^{-1} \end{bmatrix} \leq 0 \ (< 0), \tag{8}$$

where $P = P^T > 0$, $Q = Q^T > 0$, $R = R^T \geq 0$, $W = W^T \leq 0$, U, V, *and* D *are matrices of suitable sizes. Then for every matrix* $K \in \mathsf{E}$ *the matrix inequality holds:*

$$W + U^T \mathsf{D}(K) V + V^T \mathsf{D}^T(K) U + V^T \mathsf{D}^T(K) R \mathsf{D}(K) V \leq 0 \ (< 0). \tag{9}$$

Lemma 2. [13] *Suppose that* L *is symmetric matrix, the matrix* $M_1, \ldots, M_r$ *and* $N_1, \ldots, N_r$ *have corresponding sizes. Then, if for some numbers* $\varepsilon_1, \ldots, \varepsilon_r > 0$ *matrix inequality*

$$L+\sum_{i=1}^{r}\left(\varepsilon_i M_i M_i^T+\frac{1}{\varepsilon_i}N_i^T N_i\right)\le 0,$$

holds, then the inequality

$$L+\sum_{i=1}^{r}\left(M_i\Delta_i N_i+\left(M_i\Delta_i N_i\right)^T\right)\le 0,$$

is true for all $\|\Delta_i\|\le 1$ *or* $\|\Delta_i\|_F\le 1$, $i=1,\dots,r$.

We will note that Lemmas 1 and 2 are generalizations of the sufficiency statement of the adequacy criterion called Petersen's lemma on matrix uncertainty [14].

Theorem 1. *Suppose that for a positive definite matrix* $X=X^T>0$ *and for some* $\varepsilon_i>0$ $(i=1,2,3)$ *the following matrix inequalities hold:*

$$\begin{bmatrix}-G^T PG+\varepsilon_1^{-1}H_B^T H_B & D^T & B^T\\ D & -Q^{-1} & 0\\ B & 0 & -X^{-1}+\varepsilon_1 F_B F_B^T\end{bmatrix}<0, \tag{10}$$

$$\begin{bmatrix}-X+\Omega & \varepsilon_3^{-1}C_*^T H_B & C_0^T & M_*^T\\ \varepsilon_3^{-1}H_B^T C_* & -G^T PG+\varepsilon_3 H_B^T H_B & D^T & B^T\\ C_0 & D & -Q^{-1} & 0\\ M_* & B & 0 & -X^{-1}+\Theta\end{bmatrix}<0, \tag{11}$$

where $\Omega=\varepsilon_2^{-1}H_A^T H_A+\varepsilon_3^{-1}C_*^T C_*$, $\Theta=\varepsilon_2 F_A F_A^T+\varepsilon_3 F_B F_B^T$, $M_*=A+B\mathsf{D}(K_0)C$, $C_*=H_B\mathsf{D}(K_0)C$, $C_0=C+D\mathsf{D}(K_0)C$. *Then any control (2) ensures asymptotic stability of the zero state for system (1) and the general Lyapunov function* $v(x_t)=x_t^T Xx_t$.

Proof. We construct the Lyapunov function for the closed-loop system (7) as $v(x_t)=x_t^T Xx_t$. According to discrete analogue of the Lyapunov's second theorem the matrix inequality $X=X^T>0$ and negative definite first difference of the given function due to system (7) ensure asymptotic stability of the zero equilibrium state, that is with (2) it suffices that the following matrix inequality holds:

$$M^T XM-X<0. \tag{12}$$

Using property (6) of operator $\mathsf{D}(K)=(I_m-KD)^{-1}K$, we rewrite inequality (12) as

$$\left[A+\Delta A+(B+\Delta B)\left(\mathsf{D}(K_0)+\mathsf{D}(\hat{K})(I+D\mathsf{D}(K_0))\right)C\right]^T X\times$$

$$\times\left[A+\Delta A+(B+\Delta B)\left(\mathsf{D}(K_0)+\mathsf{D}(\hat{K})(I+D\mathsf{D}(K_0))\right)C\right]-X<0.$$

We rewrite last inequality as

$$M_0^T X M_0 - X + M_0^T X (B+\Delta B)\mathrm{D}(\hat{K})C_0 + C_0^T \mathrm{D}^T(\hat{K})(B+\Delta B)^T X M_0 +$$

$$+ C_0^T \mathrm{D}^T(\hat{K})(B+\Delta B)^T X (B+\Delta B)\mathrm{D}(\hat{K}) < 0,$$

where $M_0 = A + \Delta A + (B+\Delta B)\mathrm{D}(K_0)C$, $\hat{K} = G^{-1}\tilde{K}$. Here

$$\tilde{K} \in \mathrm{E} \Leftrightarrow \hat{K} \in \hat{\mathrm{E}} = \left\{K : K^T \hat{P} K \le Q\right\},$$

where $\hat{P} = G^T P G$.

We use Lemma 1 putting

$$W = M_0^T X M_0 - X\ ,\ U = (B+\Delta B)^T X M_0,\ V = C_0,\ R = (B+\Delta B)^T X (B+\Delta B)\,.$$

Then the first block inequality in (8) has the form

$$\begin{bmatrix} (B+\Delta B)^T X (B+\Delta B) - G^T P G & D^T \\ D & -Q^{-1} \end{bmatrix} < 0\,. \tag{13}$$

Inequality (4) follows from inequality (13). Then the second block inequality in (8) has the form

$$\begin{bmatrix} M_0^T X M_0 - X & M_0^T X (B+\Delta B) & C_0^T \\ (B+\Delta B)^T X M_0 & (B+\Delta B)^T X (B+\Delta B) - G^T P^{-1} G & D^T \\ C_0 & D & -Q^{-1} \end{bmatrix} \le 0\,. \tag{14}$$

We use the following well-known criterion of nonpositive (negative) definite of block matrices (Schur's lemma [15]): if $\det V \ne 0$ then

$$\begin{bmatrix} U & Z \\ Z^T & V \end{bmatrix} \le 0\ (<0) \Leftrightarrow V < 0,\ \ U - Z V^{-1} Z^T \le 0\ (<0)\,. \tag{15}$$

We see that inequality (13) can be represented as

$$\begin{bmatrix} -G^T P G & D^T & (B+\Delta B)^T \\ D & -Q^{-1} & 0 \\ B+\Delta B & 0 & -X^{-1} \end{bmatrix} < 0\,,$$

and inequality (14) can be represented as

$$\begin{bmatrix} -X & 0 & C_0^T & M_0^T \\ 0 & -G^T P^{-1} G & D^T & (B+\Delta B)^T \\ C_0 & D & -Q^{-1} & 0 \\ M_0 & B+\Delta B & 0 & -X^{-1} \end{bmatrix} < 0\,.$$

Using the structure of matrix uncertainties Δ_{At}, Δ_{Bt}, we decompose the last two inequalities:

$$\begin{bmatrix} -G^T PG & D^T & B^T \\ D & -Q^{-1} & 0 \\ B & 0 & -X^{-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ F_B \end{bmatrix} \Delta_B \begin{bmatrix} H_B & 0 & 0 \end{bmatrix} + \begin{bmatrix} H_B^T \\ 0 \\ 0 \end{bmatrix} \Delta_B^T \begin{bmatrix} 0 & 0 & F_B^T \end{bmatrix} < 0,$$

$$\begin{bmatrix} -X & 0 & C_0^T & (A+B\mathsf{D}(K_0)C)^T \\ 0 & -G^T PG & D^T & B^T \\ C_0 & D & -Q^{-1} & 0 \\ A+B\mathsf{D}(K_0)C & B & 0 & -X^{-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ F_A \end{bmatrix} \Delta_{At} \begin{bmatrix} H_A & 0 & 0 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} H_A^T \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta_A^T \begin{bmatrix} 0 & 0 & 0 & F_A^T \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ F_B \end{bmatrix} \Delta_B \begin{bmatrix} H_B \mathsf{D}(K_0)C & 0 & 0 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} C^T \mathsf{D}^T(K_0) H_B^T \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta_B^T \begin{bmatrix} 0 & 0 & 0 & F_B^T \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ F_B \end{bmatrix} \Delta_B \begin{bmatrix} 0 & H_B & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ H_B^T \\ 0 \\ 0 \end{bmatrix} \Delta_B^T \begin{bmatrix} 0 & 0 & 0 & F_B^T \end{bmatrix} < 0,$$

which is done for Lemma 2 if there are $\varepsilon_1, \varepsilon_2, \varepsilon_3 > 0$ such as

$$\begin{bmatrix} -G^T PG & D^T & B^T \\ D & -Q^{-1} & 0 \\ B & 0 & -X^{-1} \end{bmatrix} + \varepsilon_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & F_B F_B^T \end{bmatrix} + \frac{1}{\varepsilon_1} \begin{bmatrix} H_B^T H_B & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0,$$

$$\begin{bmatrix} -X & 0 & C_0^T & (A+B\mathsf{D}(K_0)C)^T \\ 0 & -G^T PG & D^T & B^T \\ C_0 & D & -Q^{-1} & 0 \\ A+B\mathsf{D}(K_0)C & B & 0 & -X^{-1} \end{bmatrix} + \varepsilon_2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F_A F_A^T \end{bmatrix} +$$

$$+ \frac{1}{\varepsilon_2} \begin{bmatrix} H_A^T H_A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \varepsilon_3 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F_B F_B^T \end{bmatrix} +$$

$$+\frac{1}{\varepsilon_3}\begin{bmatrix} C^T \mathsf{D}^T(K_0)H_B^T H_B \mathsf{D}(K_0)C & C^T \mathsf{D}^T(K_0)H_B^T H_B & 0 & 0 \\ H_B^T H_B \mathsf{D}(K_0)C & H_B^T H_B & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} < 0 .$$

We get inequalities equivalent to conditions of the form (10) and (11) under which matrix inequality (12) holds. These conditions ensure asymptotic stability for the zero state of the closed-loop system (7) for any control (2).

This completes the proof of the theorem.

Bounds on the quadratic quality criterion under uncertainty conditions. Consider a control system (1), (2) with quadratic quality functional

$$J_u(x_0) = \sum_{t=0}^{\infty} \varphi_t \, , \; \varphi_t = \begin{bmatrix} x_t^T & u_t^T \end{bmatrix} \Phi \begin{bmatrix} x_t^T \\ u_t^T \end{bmatrix}, \; \Phi = \begin{bmatrix} S & N \\ N^T & R \end{bmatrix} > 0 \, , \tag{16}$$

where x_0 is initial vector, $S = S^T > 0$, $R = R^T > 0$, and N given constant matrices.

We need to describe the set of controls (2) that would provide asymptotic stability for the state $x_t \equiv 0$ of system (1) and a bound

$$J_u(x_0) \le \omega \, , \tag{17}$$

where $\omega > 0$ is some maximal admissible value of the functional. When solving this problem, we still use the quadratic Lyapunov function $v(x_t) = x_t^T X x_t$ under constraint $x_0^T X x_0 \le \omega$. Under assumptions (2) and (4) values of $\mathsf{D}(K)$, $\mathsf{D}(K_0)$, and $\mathsf{D}(\hat{K})$ are defined, where $\hat{K} = G^{-1}\tilde{K}$, $G = I_m - K_0 D$. Here the closed-loop system can be represented as (7), and the first difference v of function due to system (7) and the summable function in (16) have the form

$$v(x_{t+1}) - v(x_t) = x_t^T (M^T X M - X) x_t \, , \; \varphi_t = x_t^T L^T \Phi L x_t \, ,$$

where $L^T = \begin{bmatrix} I_n & C^T \mathsf{D}^T(K) \end{bmatrix}$, $K = K_0 + \tilde{K}$.

We now require that together with (4) the following inequality holds:

$$v(x_{t+1}) - v(x_t) \le -\varphi_t \, . \tag{18}$$

For this it suffices that the following matrix inequality holds:

$$M^T X M - X + L^T \Phi L < 0 \, . \tag{19}$$

Then the zero solution $x_t \equiv 0$ of system (1) is asymptotically stable and together with (18) we get an upper bound on the functional:

$$J_u(x_0) \le \sum_{t=0}^{\infty} \left[v(x_t) - v(x_{t+1}) \right] = x_0^T X x_0 \le \omega \, . \tag{20}$$

Using property (6) of operator Δ , we rewrite inequality (19) as

$$W + U^T \mathsf{D}(\hat{K})V + V^T \mathsf{D}^T(\hat{K})U + V^T \mathsf{D}^T(\hat{K})\hat{R}\mathsf{D}(\hat{K})V < 0 \quad , \tag{21}$$

where $W = M_0^T X M_0 - X + L_0^T \Phi L_0$, $U = (B + \Delta B)^T X M_0 + N^T + R\mathsf{D}(K_0)C$, $V = C_0 = C + D\mathsf{D}(K_0)C$, $L_0^T = \begin{bmatrix} I_n & C^T \mathsf{D}^T(K_0) \end{bmatrix}$, $\hat{R} = R + (B + \Delta B)^T X (B + \Delta B)$.

Here

$$\tilde{K} \in \mathsf{E} \Leftrightarrow \hat{K} \in \hat{\mathsf{E}} = \left\{ K : K^T \hat{P} K \le Q \right\},$$

where $\hat{K} = G^{-1}\tilde{K}$, $\hat{P} = G^T P G$.

Applying Lemma 1, relations (18)-(21), and Lemma 2, we arrive at the following result.

Theorem 2. *Suppose that for a positive definite matrix* $X = X^T > 0$ *and for some* $\varepsilon_i > 0$ $(i = 1, 2, 3)$ *the following matrix inequalities hold:*

$$\begin{bmatrix} R - G^T P G + \varepsilon_1^{-1} H_B^T H_B & D^T & B^T \\ D & -Q^{-1} & 0 \\ B & 0 & -X^{-1} + \varepsilon_1 F_B F_B^T \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} -X + \Omega & N_*^T & C_0^T & M_*^T \\ N_* & R - G^T P G + \varepsilon_3 H_B^T H_B & D^T & B^T \\ C_0 & D & -Q^{-1} & 0 \\ M_* & B & 0 & -X^{-1} + \Theta \end{bmatrix} < 0, \tag{23}$$

where $\Omega = L_0^T \Phi L_0 + \varepsilon_2^{-1} H_A^T H_A + \varepsilon_3^{-1} C_*^T C_*$, $\Theta = \varepsilon_2 F_A F_A^T + \varepsilon_3 F_B F_B^T$, $M_* = A + B\mathsf{D}(K_0)C$, $N_* = N^T + R\mathsf{D}(K_0)C$, $C_* = H_B \mathsf{D}(K_0)C$. *Then any control (2) ensures asymptotic stability of the zero state for system (1), the general Lyapunov function* $v(x_t) = x_t^T X x_t$, *and a bound on the functional (17).*

Based on Theorem 2 and its corollaries, we can formulate the following optimization problem for system (1): *minimize* $\omega > 0$ *under constraints* (22), (23).

The results of Theorems 1 – 2 can be generalized in case when

$$\Delta A(t) = \sum_{i=1}^{r} F_A^{(i)} \Delta_{At}^{(i)} H_A^{(i)}, \quad \Delta B(t) = \sum_{i=1}^{r} F_B^{(i)} \Delta_{Bt}^{(i)}(t) H_B^{(i)} .$$

Numerical experiment. Consider a control system for a double oscillator. It is system of two solids that connected by a spring and slide without a friction along of horizontal rod. This system is defined with two linear differential equations of order two, or, in vector-matrix form [13]:

$$\dot{x} = (A_c + \Delta A(t))x + B_c u , \tag{24}$$

where

$$A_c+\Delta A(t)=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -\dfrac{k_0}{m_1} & \dfrac{k_0}{m_1} & 0 & 0\\ \dfrac{k_0}{m_2} & -\dfrac{k_0}{m_2} & 0 & 0\end{bmatrix}+F_A\Delta(t)H_A,\ B_c=\begin{bmatrix}0\\0\\1\\0\end{bmatrix},\ F_A=\begin{bmatrix}0\\0\\-\delta\\ \delta\end{bmatrix},$$

$$H_A=\begin{bmatrix}1 & -1 & 0 & 0\end{bmatrix},\ x=\begin{bmatrix}x_1 & x_2 & \dot{x}_1 & \dot{x}_2\end{bmatrix}^T.$$

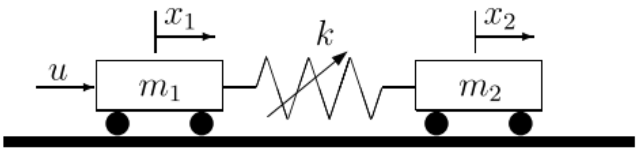

Figure 1. A two-masse mechanical system

Here x_1 and $\dot{x}_1$ are coordinate and velocity respectively for the first solid, x_2 and $\dot{x}_2$ are coordinate and velocity respectively for the second solid, m_1 and m_2 are masses of the first and second solids respectively. We define a stiffness coefficient as variable periodic function of time $k=k_0+\delta\Delta(t)$, where $\Delta(t)=\sin(\varpi t)$, $\delta<<1$ is the amplitude of harmonic oscillations, and ϖ is the frequency parameter.

We will make the discrimination of system (24) in the form:

$$x_{t+1}=(A+\Delta A_t)x_t+Bu_t,\ \ A=I_4+\tau A_c,\ \ \Delta A_t=\Delta A(\tau t),\ \ B=\tau B_c,\ \ t=0,1,2,..., \qquad (25)$$

where $x_t=x(t\tau)$, $u_t=u(t\tau)$, τ is the pitch of discrimination. Let $\tau=0{,}0005$, $m_1=1$, $m_2=1$, $k_0=1$, $\delta=0{,}01$, $\Delta(t)=\sin(t/5)$.

We assume that the output vector

$$y_t=Cx_t+Du_t=\begin{bmatrix}\dot{x}_{1t}+u_t\\ x_{2t}\end{bmatrix},\ C=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0\end{bmatrix},\ D=\begin{bmatrix}1\\0\end{bmatrix}$$

can be measured.

We find control in the form static output feedback $u_t=Ky_t$, where $K=\begin{bmatrix}k_1 & k_2\end{bmatrix}=K_0+\tilde{K}$. We find the vector $K_0=\begin{bmatrix}1{,}6938 & 0{,}1089\end{bmatrix}$ that ensures asymptotic stability for system $x_{t+1}=M_0x_t$, $M_0=A+B\mathrm{D}(K_0)C$. Here the spectrum $\sigma(M_0)=\{0{,}9989;0{,}9999\pm0{,}0005i;0{,}9999\}$ places in the middle of unit disk [12]. The behavior of solutions of system with matrix uncertainty (25) with control $u_t=K_0y_t$ and initial vector $x_0=\begin{bmatrix}1 & 0 & -1 & 2\end{bmatrix}^T$ is shown on Fig. 2.

For demonstration of Theorem 2 we define a matrix functional (16): $S=0{,}1I_4$, $R=0{,}01$, $N^T=\begin{bmatrix}0{,}01 & 0 & 0 & 0{,}01\end{bmatrix}$. Using the Matlab suite, we find $P=0{,}0009$ and positive definite matrices

$$Q = 10^{-8}\begin{bmatrix} 0,2267 & -0,0013 \\ -0,0013 & 0,2308 \end{bmatrix}, \ X = 10^{3}\begin{bmatrix} 1,6556 & -1,6556 & 0,0005 & -0,0008 \\ -1,6556 & 1,6556 & -0,0005 & 0,0008 \\ 0,0005 & -0,0005 & 0,0000 & -0,0000 \\ -0,0008 & 0,0008 & -0,0000 & 0,0000 \end{bmatrix},$$

that satisfy the inequalities (22), (23) for $\varepsilon_1 = 0{,}01$.

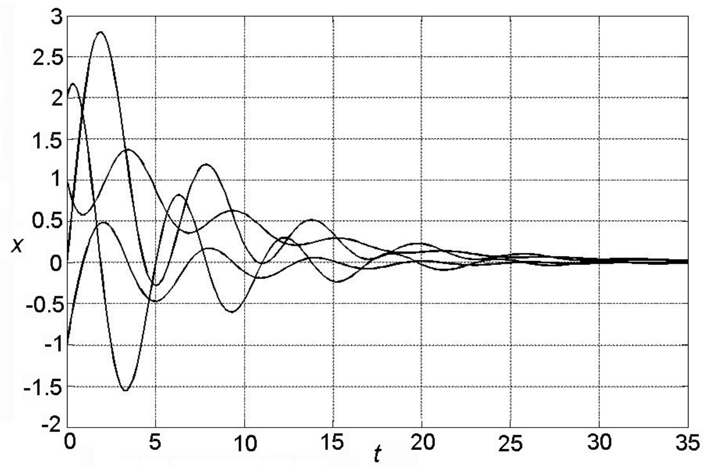

Figure 2. System behavior with control $u_t = K_0 y_t$

Thus, for all values of the vector of feedback amplification coefficients $K = K_0 + \tilde{K}$ from a closed region bounded by the ellipse $\mathrm{E} = \{K : KQ^{-1}K^T \le P^{-1}\}$ (Fig. 3), the motion of the system of two solids in a neighborhood of the zero state is asymptotically stable. Here $v(x_t) = x_t^T X x_t$ is a general Lyapunov function, and the value of the given quality functional does not exceed $v(x_0) = 1651{,}3$.

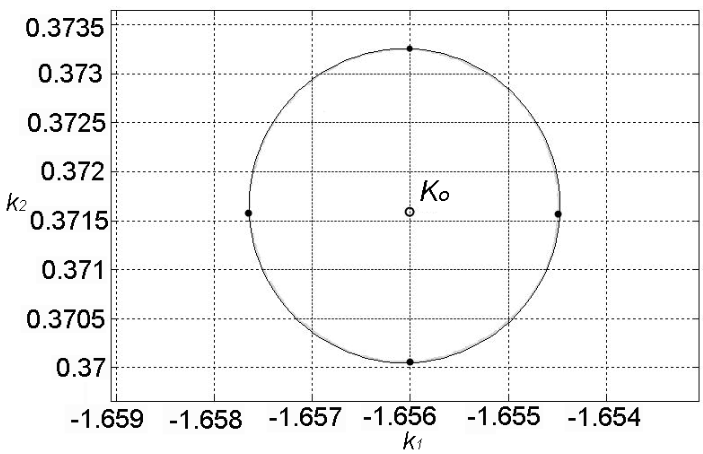

Figure 3. Region of feedback amplification coefficients

Conclusions. In this work, we have proposed new methods of robust stability analysis and optimization of linear difference systems with static output feedback. Here values of unknown matrix coefficients are defined by restrictions on norm of matrix uncertainties and the measurable output vector contains components of both the system state and the control.

Practical implementation of the proposed methods is related to solving differential or algebraic matrix inequalities. An important characteristic feature that distinguishes matrix inequalities that we have found from known ones is the possibility to construct an ellipsoid of stabilizing matrices for the feedback amplification coefficients, general quadratic Lyapunov function, and also bounds on the quadratic quality functional for linear control systems with the considered matrix uncertainties.

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РОБАСТНА СТІЙКІСТЬ І ОЦІНКА ФУНКЦІОНАЛА ЯКОСТІ ЛІНІЙНИХ ДИСКРЕТНИХ СИСТЕМ З МАТРИЧНИМИ НЕВИЗНАЧЕНОСТЯМИ

Андрій Алілуйко; Валерій Єрьоменко

Тернопільський національний економічний університет, Тернопіль, Україна

Резюме. *Розроблено нові методи аналізу робастної стійкості та оптимізації дискретних систем керування із зворотним зв'язком. Для лінійних керованих систем з невизначеними матричними коефіцієнтами та зворотним зв'язком за вимірюваним виходом формулюються достатні умови стійкості нульового стану із спільною функцією Ляпунова. Запропоновано розв'язання задачі робастної стабілізації та оцінки квадратичного критерію якості лінійних дискретних систем з матричними невизначеностями. Наведено приклад стабілізації двомасової механічної системи.*

Ключові слова: *робастна стійкість, матрична невизначеність, дискретна система, зворотний зв'язок по виходу.*

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