Міністерство освіти і науки України
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імені Івана Пулюя

навчально–методична література

кафедра
світлотехніки та
електротехніки

Electrical Engineering

LECTURES

Ternopil 2017

«Electrical Engineering. Lectures.»

Укладач: Н.А. Куземко. Тернопіль: ТНТУ, 2017. – 58 с.

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Редагування англійською мовою: к.п.н., доц. Кухарська В.Б.

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Затверджено на засіданні кафедри світлотехніки та електротехніки Тернопільського національного технічного університету імені Івана Пулюя.
Протокол № 6 від 28 лютого 2017 року

Схвалено й рекомендовано до друку методичною радою електромеханічного факультету Тернопільського національного технічного університету імені Івана Пулюя.
Протокол № 6 від 20 березня 2017 року

Посібник складено відповідно до робочих програм курсів “Електротехніка, електроніка та основи МПТ“, “Електротехніка в будівництві”, “Теорія електричних та магнітних кіл”.

Part 1. Direct current circuits.

Sources and consumers.

Electrical circuit is the multitude of devices designed for transforming, distribution and conversion of electrical energy, while the processes which are taking places in these devices can be described by the concepts of current, voltage and electromotive force (e.m.f.).

The simplest electrical circuit contains three main elements: electrical source (active element), consumer (passive element) and the wires. Besides, the circuit can have also additional elements: measuring devices, switches, fuses, contactors, etc.

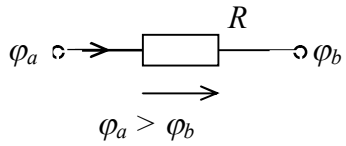

Fig.1.1

Electrical power is transformed into heating, mechanical energy, etc. at the *consumers*. The measure of this transformation is resistance R (fig.1.1). You can see the directions of the electrical values at fig.1.1.

Ohm's law for this element is as follow $V = RI$ or $I = GV$, where R - is resistance, $G = 1/R$ - is conductivity. The power on resistive element is $P = RI^2 = GV^2$.

Heating, mechanical energy, etc. is transformed into electrical power at the *electrical sources*. The measure of this transformation is electromotive force (e.m.f.) E (fig.1.2). You can see the directions of the electrical values at fig.1.2.

The *ideal electrical source* (without losses) is characterized only by E. The

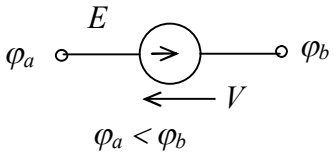

Fig.1.2

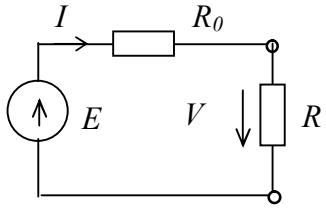

Fig.1.3

power on the electrical source is $P = EI$.

The *real electrical source* has losses and is characterized by E and R_0 (internal resistance), which reflects the losses. The simplest electrical circuit is shown at fig.1.3. For this circuit:

$I = E/(R_0 + R)$, then $V = E - R_0 I$, $V = RI$, $R_0 I + RI = E$.

We can represent real electrical circuit by two substitution schemes: serial (fig.1.4) and parallel (fig.1.6). The external volt-ampere characteristic (fig.1.5) *V(I)* is the main characteristic of the source. Its analytical expression is $V = E - R_0 I$. At fig.1.5 solid line indicates the characteristic of real source, dashed line - the

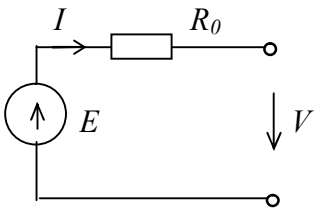

Fig.1.4

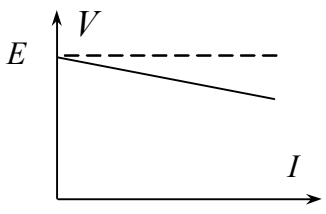

Fig.1.5

characteristic of ideal source. Boundary points of this characteristic correspond to the source boundary modes – open circuit (idle) mode (without loading), when $I=0$, $V=E=V_{OC}$ and short circuit mode, when $V=0$, $I=I_{SC}$. The external characteristic of ideal source $V=E$ is represented by dashed line at fig.1.5.

Parallel substitution scheme (fig.1.6) consists of ideal current source J and internal conductivity G_0, which characterizes the losses. The external characteristic of real source (fig.1.7) is described by the equation $I=J-G_0U$. The external characteristic of ideal source $I=J$ is represented by dashed line at fig.1.7.

Serial and parallel schemes are equivalent, it means you can transform one into another using such formulas:

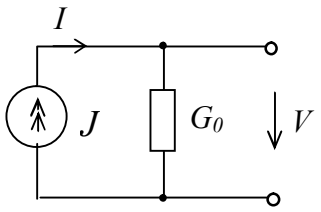

Fig.1.6

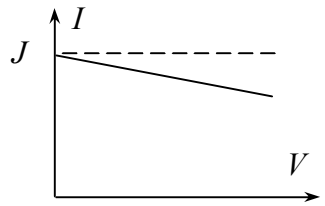

Fig.1.7

$$E=G_0J,\ R_0=1/G_0,\ J=E/R_0,\ G_0=1/R_0.$$

The efficiency factor of the source characterizes the efficiency of energy transforming from the source to consumer:

$$\eta=\frac{P_R}{P_E}=\frac{VI}{EI}=\frac{V}{E},\quad V=\eta E,$$

where P_R - is a consumer power, P_E - is a source power.

We can also write down the efficiency factor using the elements parameters:

$$\eta=\frac{P_R}{P_R+\Delta P}=\frac{RI^2}{RI^2+R_0I^2}=\frac{R}{R_0+R}=\frac{1}{1+R_0/R},$$

where ΔP - are power losses.

There are three main *electrical circuit modes:* nominal, operating and boundary.

The nominal mode is the best mode for the working device, the device nominal parameters are shown in its technical passport ($I_{NOM}, V_{NOM}, P_{NOM}$).

Operating mode is a mode, where the deviation from the nominal parameters is not big.

Boundary modes are: open circuit or idle (non-working) and a short circuit (emergency) modes. For the open circuit (o.c.) mode $R=\infty$, then using the scheme at fig.1.3, we can write down:

$$I=\frac{E}{R_0+R}=\frac{E}{R_0+\infty}=0,\ V=E-R_0I=E,\ \eta=1.$$

For short circuit mode (s.c.) $R=0$, then using the scheme at fig.1.3, we can write down:

$$V=RI,\ V=0,\ I=E/R_0=I_{SC},\ \eta=0.$$

The methods of open circuit and short circuit experiments can be used for

defining the parameters of the source (E,R_0): $V_{OC}=E,\ R_0=E/I_{SC}$. The experiment of s.c. mode is provided at low voltage.

Electrical source operating modes:

- voltage generator, when the voltage at the clamps of the source practically does not depend on the current, thus $V \approx E$, and this mode is close to o.c. In this mode $R_0 I << RI$ (fig.1.4), that's why the condition of it is $R_0 << R$ and $\eta \approx 1$. This is the main operating mode of electrical engineering devices.

- current generator, when the current at the clamps of the source practically does not depend on the voltage, thus $I \approx J$, and this mode is close to to s.c. In this mode $G_0 U << GU$ (fig.1.6), that's why the condition of it is $G_0 << G$ ($R_0 >> R$).

-balanced mode – the maximum power $P = R I^2$ is transferred from the source to the consumer at this mode? $I = E/(R_0+R)$, and $P = RE^2/(R_0+R)^2$ at this mode.

The condition of this mode comes out from the expression $dP/dR = 0$, that means $R_0 = R$ and $\eta = 0.5$. This mode is used in electronics.

Connections of elements

There are two types of elements connections in electrical circuits, they are simple and complex. The major difference between those two types is that we know the directions of currents before we calculate the circuit with simple connection and don't know the directions of currents at the circuits with complex connections, that's why we choose them arbitrarily.

There are three types of *simple* connection: serial, parallel and mixed.

When the elements are connected in *serial* (fig.1.8), the same current I is flowing through them. The total resistance of serial connection is $R = \Sigma R_n$.

The input voltage (fig.1.8)

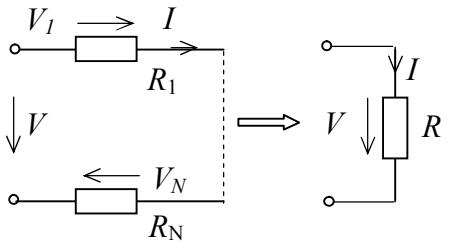

Fig.1.8

$$V = V_1 + \ldots + V_N = \Sigma V_N = \Sigma R_N I .$$

The power of this circuit

$$P = VI = \Sigma R_N II = \Sigma R_N I^2 = \Sigma P_N ,$$

where $P = VI$ – the power of the source, ΣP_N – the power of the consumers.

When the elements are connected in *parallel* (fig.1.9), the same voltage V is applied to them. The total conductivity of parallel connection is $G = \Sigma G_n$.

The total current of the circuit (fig.1.9):

$$I = I_1 + \ldots + I_N = \Sigma I_N = \Sigma G_N V .$$

The power of this circuit:

$$P = VI = \Sigma G_N VV = \Sigma G_N V^2 = \Sigma P_N .$$

For two elements connected in parallel:

$$G = G_1 + G_2 = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2},$$

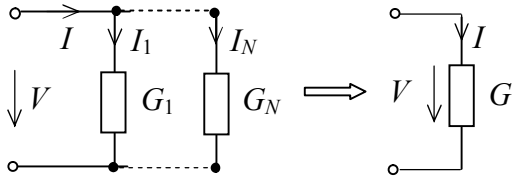

Fig.1.9

$$R = \frac{1}{G} = \frac{R_1 R_2}{R_1 + R_2}.$$

The circuit with two elements connected in serial (fig.1.10) can be used as *voltage divider*.

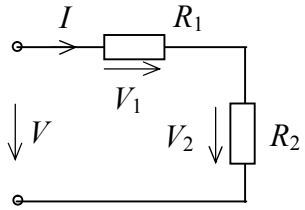

Fig.1.10

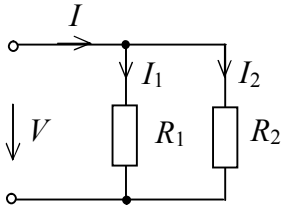

Fig.1.11

$$I = \frac{V}{R_1 + R_2}, \qquad V_1 = R_1 I = V \frac{R_1}{R_1 + R_2}, \qquad V_2 = R_2 I = V \frac{R_2}{R_1 + R_2}.$$

The circuit with two parallel connected elements (fig.1.11) can be used as *current divider.*

$$V = I \frac{R_1 R_2}{R_1 + R_2}, \; I_1 = V / R_1 = I \frac{R_2}{R_1 + R_2}, \; I_2 = V / R_2 = I \frac{R_1}{R_1 + R_2}.$$

We can replace the *mixed* (serial-parallel) connection (fig.1.12) by one equivalent (total) resistance R:

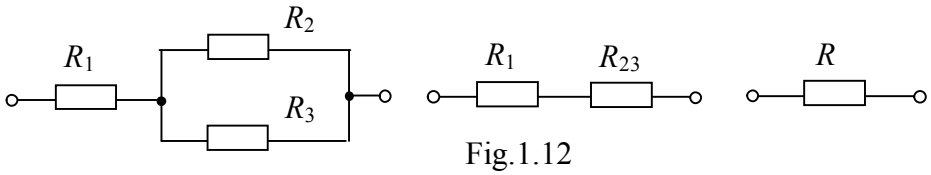

Fig.1.12

$$R_{23} = R_2 R_3 / (R_2 + R_3), \quad R = R_1 + R_{23}.$$

We can also replace the *mixed* (parallel-serial) connection (fig.1.13) by one equivalent (total) resistance R:

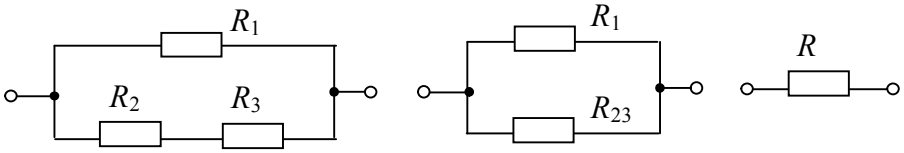

Fig.1.13

$$R_{23} = R_2 + R_3, \qquad R = R_1 R_{23} / (R_1 + R_{23})$$

The *complex* connections are DELTA (fig.1.14) and WYE (fig.1.15).

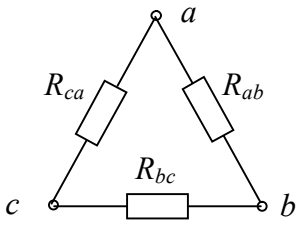

Fig.1.14

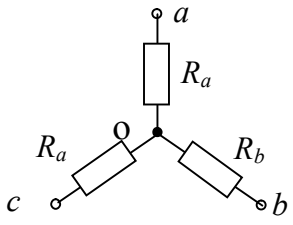

Fig.1.15

We can know real directions of the currents only after calculation. We can also transform DELTA into WYE using such expressions:

$$R_a = \frac{R_{ab}R_{ca}}{R_{ab}+R_{bc}+R_{ca}}, \qquad R_b = \frac{R_{ab}R_{bc}}{R_{ab}+R_{bc}+R_{ca}}, \qquad R_c = \frac{R_{ca}R_{bc}}{R_{ab}+R_{bc}+R_{ca}}.$$

We use the *simplification method* to calculate the circuits with one source. To use this method we must:

- simplify the circuit to one equivalent resistance;
- calculate the total current by using Ohm's law;
- revert back the circuit and calculate the branch currents and voltages across the elements;
- verify the calculation by using the power balance equation.

The calculation of electrical circuits with several sources.

We can use several methods, which are based on Kirchhoff's laws.

Kirchhoff's first law states that the sum of the currents entering the node is equal to the sum of the currents leaving the node $\sum I_n = 0$ (the algebraic sum of the currents in the node is equal to zero).

Kirchhoff's second law states that the algebraic sum of all voltages across passive elements around a loop is equal the algebraic sum of electro-motive forces around the same loop $\Sigma R_n I_n = \Sigma E_n$.

Branch of the circuit is the part of the circuit with the same current, it may be consisted from one or several elements connected in serial.

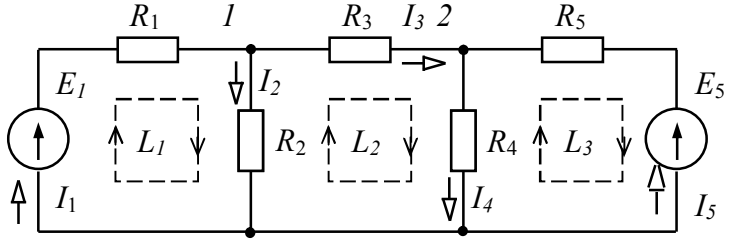

Fig.1.16

Node is the place where three or more branches are connected.

Loop is any closed path around the circuit.

Kirchhoff's laws method.

Let`s suppose the circuit has p branches and q nodes. There'll be p unknown currents. We must solve the system of p equations to find them.

First, you have to choose the directions of branch currents arbitrarily and mark them at the scheme, then mark the nodes and the loops. After this, it is necessary to write down $q-1$ nodes equations according to Kirchhoff's first law and $p-q-1$ loop equations according to the Kirchhoff's second law.

After the equations system is solved, some currents may have sign "-", it means that the real directions of that current is opposite to the one we have chosen at the beginning.

Let`s write down the equations system for the scheme at fig.1.16. There are 5 branches $p=5$ and 3 nodes $q=3$ here.

The equations according to Kirchhoff's first law ($q-1=2$) for the nodes *1* and

2:

$$\begin{array}{l} \text{“1“ } I_1 = I_2 + I_3 \\ \text{“2“ } I_3 + I_5 = I_4 \end{array} \qquad (1)$$

The equations according to Kirchhoff's second law ($p-q-1=3$) for the loops L_1, L_2, L_3 (we choose the directions along the loops clockwise , if the directions of our bypass and the voltage or e.m.f. are the same, we denominate it with "+", if opposite with "-").

$$\begin{array}{l} \text{“ L}_1\text{“ } + R_1 I_1 + R_2 I_2 = +E_1 \\ \text{“ L}_2\text{“} - R_2 I_2 + R_3 I_3 + R_4 I_4 = 0 \\ \text{“ L}_3\text{“} - R_4 I_4 - R_5 I_5 = -E_5 \end{array} \qquad (2)$$

So, the equation system according this method will be:

$$\begin{cases} +I_1 - I_2 - I_3 = 0 \\ +I_3 - I_4 + I_5 = 0 \\ +R_1 I_1 + R_2 I_2 = +E_1 \\ -R_2 I_2 + R_3 I_3 + R_4 I_4 = 0 \\ -R_4 I_4 - R_5 I_5 = -E_5 \end{cases} \qquad (3)$$

After solving this system we get the unknown branch currents.

We apply the equation of power balance to verify our calculations: the total power of the sources should be equal to the total power of the consumers $\Sigma P_R = \Sigma P_E$. The total power of the sources $\Sigma P_E = \Sigma E_n I_n = E_1 I_1 + E_5 I_5$. The total power of the consumers $\sum P_R = \sum R_n I_n^2 = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2 + R_4 I_4^2 + R_5 I_5^2$.

Loop currents method.

This method has less equations than previous and is based on the Kirchhoff's

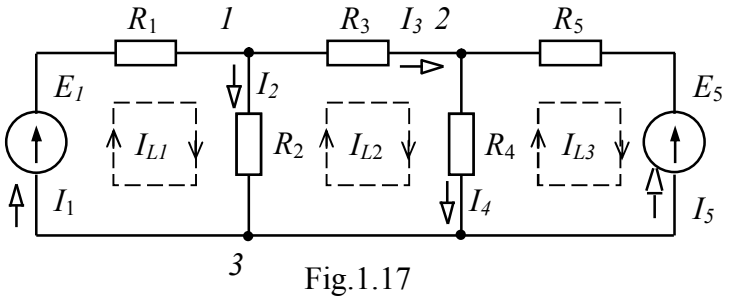

Fig.1.17

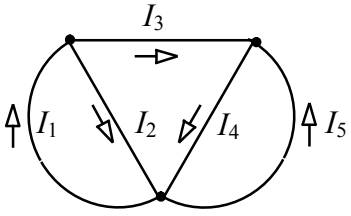

Fig.1.18

second law. Let's suppose that we have three loop currents I_{L1}, I_{L2}, I_{L3} at circuit (fig.1.17), the directions of these currents we choose arbitrarily. Then we can write down branch current by using loop currents: $I_1 = I_{L1}$, $I_2 = I_{L1} - I_{L2}$, $I_3 = I_{L2}$, $I_5 = -I_{L3}$, $I_4 = I_{L2} - I_{L3}$.

We have to substitute these expressions in the equations of Kirchhoff's second law:

$$\text{“ L}_1\text{“ } + R_1 I_1 + R_2 I_2 = +E_1$$
$$\text{“ L}_2\text{“} - R_2 I_2 + R_3 I_3 + R_4 I_4 = 0$$
$$\text{“ L}_3\text{“} - R_4 I_4 - R_5 I_5 = -E_5$$

We get the following:

$$\begin{cases} (R_1 + R_2) I_{L1} - R_2 I_{L2} = E_1 \\ -R_2 I_{L1} + (R_2 + R_3 + R_4) I_{L2} - R_4 I_{L3} = 0 \\ -R_4 I_{L2} + (R_4 + R_5) I_{L3} = -E_5 \end{cases} \quad (4)$$

Let's mark:

$R_{11} = R_1 + R_2$, $R_{22} = R_2 + R_3 + R_4$, $R_{33} = R_4 + R_5$ - it'll be individual resistances of the loops, which are equal to the sum of all the resistances of the loop;

$R_{12} = R_{21} = R_2$, $R_{13} = R_{31} = 0$, $R_{23} = R_{32} = R_4$ - mutual resistances of the loops, the resistances of the branches which are mutual for the respective loops;

$E_{L1} = E_1$, $E_{L2} = 0$, $E_{L3} = -E_5$ - loops e.m.f., is equal to the algebraic sum of the electromotive forces of the loops.

Using these markings, system (4) looks like (5), that can be used for any circuit with three independent loops:

$$\begin{cases} +R_{11} I_{L1} - R_{12} I_{L2} - R_{13} I_{L3} = E_{L1} \\ -R_{21} I_{L1} + R_{22} I_{L2} - R_{23} I_{L3} = E_{L2} \\ -R_{31} I_{L1} - R_{32} I_{L2} + R_{33} I_{L3} = E_{L3} \end{cases} . \quad (5)$$

Nodal potential method

This method has less equations than previous one and is based on Kirchhoff's first law. Let's analyze the circuit on fig.1.19. There are two independent nodes a,b. Try to suppose that the potential of the basic (dependent) node is equal to zero, so the potentials of other nodes are marked at the scheme as φ_a, φ_b (fig.1.19).

We can also write down the branch currents using node potentials:

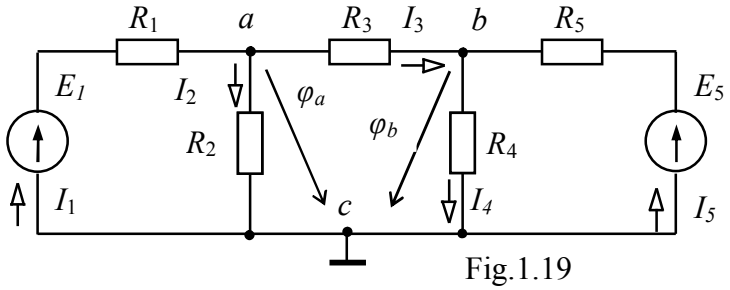

Fig.1.19

$$\varphi_a = E_1 - R_1 I_1, \quad I_1 = \frac{(E_1 - \varphi_a)}{R_1} = (E_1 - \varphi_a) G_1,$$

$$\varphi_a = R_2 I_2,\ I_2 = \frac{\varphi_a}{R_2} = \varphi_a G_2, \qquad \varphi_b = R_4 I_4,\ I_4 = \frac{\varphi_b}{R_4} = \varphi_b G_4,$$

$$\varphi_a - \varphi_b = R_3 I_3,\ I_3 = \frac{(\varphi_a - \varphi_b)}{R_3} = (\varphi_a - \varphi_b) G_3,$$

$$\varphi_b = E_5 - R_5 I_5,\ I_5 = \frac{(E_5 - \varphi_b)}{R_5} = (E_5 - \varphi_b) G_5 .$$

Let's substitute these expressions into the equations for the nodes a,b

$$+I_1 - I_2 - I_3 = 0$$
$$+I_3 - I_4 + I_5 = 0\text{'}$$

we get

$$\begin{cases} (G_1 + G_2 + G_3)\varphi_a - G_3 \varphi_b = G_1 E_1 \\ -G_3 \varphi_a + (G_3 + G_4 + G_5)\varphi_b = G_5 E_5 \end{cases}. \qquad (6)$$

Let's mark:

$G_{11} = G_1 + G_2 + G_3,\ G_{22} = G_3 + G_4 + G_5$ - the individual conductivities of the nodes, it's the sum of the branch conductivities which coming in the node;

$G_{12} = G_{21} = G_3$ - the mutual conductivities of the nodes, the conductivity of the branch, which connects respective nodes;

$J_a = G_1 E_1,\ J_b = G_5 E_5$ - the algebraic sum of the currents of current sources, which are flowing in the respective nodes. If the current J of the source flows in the node, we mark it by the sign "+", when it flows out – with sign "-".

Using these markings, system (6) looks like (7), that can be used for any circuit with two independent nodes:

$$\begin{cases} G_{11}\varphi_a - G_{12}\varphi_b = J_a \\ -G_{12}\varphi_a + G_{22}\varphi_b = J_b \end{cases}. \qquad (7)$$

Two nodes method.

This method is used for calculating the circuits with only two nodes and several parallel branches. The example of such circuit is on fig.1.20. This method is also based on the Kirchhoff's first law and is partly the method of nodal potentials. First of all, we calculate the inter-node voltage $V = \Sigma G_n E_n / \Sigma G_n$, where G_n - conductivity of *n* branch, E_n - e.m.f.of *n* branch. For the circuit on fig.1.20 it'll be

$$V_{ab} = \frac{G_1 E_1 + G_2 E_2}{G_1 + G_2 + G_3}.$$

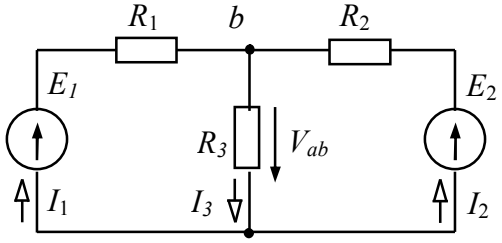

Fig.1.20

Then we calculate the branch currents using such expressions:

$$V_{ab} = R_3 I_3,\ I_3 = \frac{V_{ab}}{R_3} = V_{ab} G_3,$$

$$V_{ab} = E_1 - R_1 I_1,\quad I_1 = \frac{(E_1 - V_{ab})}{R_1} = (E_1 - V_{ab}) G_1,$$

$$V_{ab} = E_2 - R_2 I_2,\quad I_2 = \frac{(E_2 - V_{ab})}{R_2} = (E_2 - V_{ab}) G_2 .$$

The superposition method.

We can use this method when the e.m.f. of one source is changed. The method based on the superposition principle, means that every e.m.f. acts in the circuit independently. So, the calculation of one circuit (fig.1.20) with two sources, for example, can be reduced to the calculation of two circuits with one source (fig.1.21, 1.22).

According to this method, we must calculate two partial circuits with partial currents. We have only e.m.f. E_1 in the first partial circuit (fig.1.21).

The total resistance of this circuit: $R' = R_1 + \dfrac{R_2 R_3}{R_2 + R_3}$.

The partial branches currents: $I'_1 = E_1 / R'$, $I'_2 = I'_1 \dfrac{R_3}{R_2 + R_3}$, $I'_3 = I'_1 \dfrac{R_2}{R_2 + R_3}$.

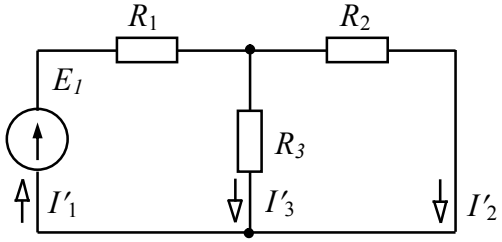

Fig.1.21

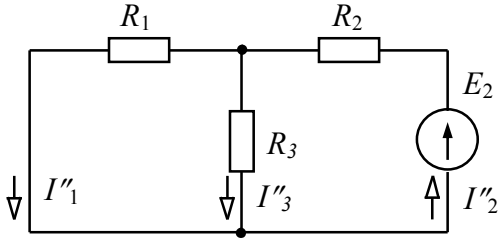

Fig.1.22

We have only e.m.f. E_2 in the second partial circuit (fig.1.22).

The total resistance of this circuit: $R'' = R_2 + \dfrac{R_1 \cdot R_3}{R_1 + R_3}$.

The partial branches currents: $I''_2 = E_2 / R''$, $I''_1 = I''_2 \dfrac{R_3}{R_2 + R_3}$, $I''_3 = I''_2 \dfrac{R_1}{R_1 + R_3}$.

Then we have the real branch currents as an algebraic sum of the respective partial currents (fig.1.20):

$$I_1 = I'_1 - I''_1,\ I_2 = I''_2 - I'_2,\ I_3 = I''_3 + I'_3.$$

Equivalent generator method.

The method is used when it is necessary to calculate the current of only one branch of the circuit (for example it is varying resistor or non-linear element in this branch). We select the branch with unknown current (e.g. I_3) from the circuit on fig.1.20 and the rest of the circuit is replaced by the equivalent generator (fig.1.23) with parameters E_{eqv} - equivalent e.m.f., which is equal to the open circuit voltage on

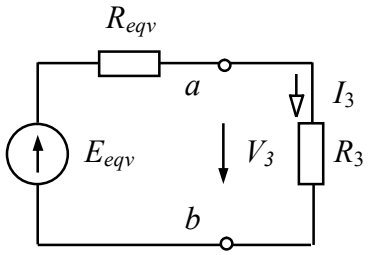

Fig.1.23

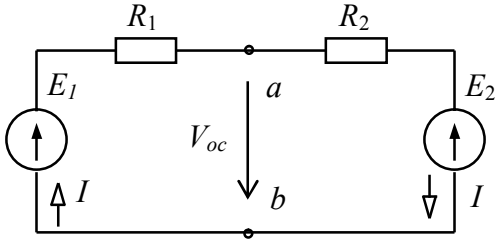

Fig.1.24

the clamps of an open branch *ab* and R_{eqv} – equivalent resistance, which is equal to the input resistance of the circuit in respect to the open branch *ab*. The problem is to calculate the parameters of equivalent generator E_{eqv} and R_{eqv}. For the circuit at fig.1.20 $R_{екв} = \frac{R_1 R_3}{R_1 + R_3}$. Then we can calculate E_{eqv} using fig.1.24 $V_{OC} = E_{eqv} = E_1 - R_1 I$, where $I = \frac{E_1 - E_2}{R_1 + R_3}$.

According to the fig.1.23 we calculate unknown current $I_3 = E_{eqv} / (R_{eqv} + R_2)$.

Direct currents non-linear circuits.

Non-linear circuits consist of one or more non-linear elements. We call an element non-linear when its resistance is not constant and depends on voltage, current, temperature, light, etc. The volt-ampere characteristic (VAC) $V(I)$ is the main characteristic of non-linear element and it's non-linear (fig.1.25).

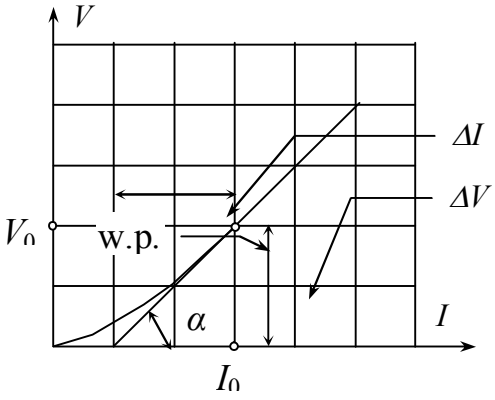

Fig.1.25.

There are non-controlled and controlled non-linear elements. Non-controlled elements have two clamps (lamps, diods), controlled elements have three or more clamps (transistors, thyristors). VAC of non-linear elements may be symmetrical or non-symmetrical. If the resistance of the element doesn't depend on the direction of the current and the polarity of voltage then the characteristic is symmetrical. We can present VAC by graphs, tables or formulas $V(I)$.

Non-linear circuits can be calculated by analytical or graph methods. If we use graph method we define the voltage and current of the circuit using VACs of the elements. We can use Ohm's and Kirchhoff's laws as well. Analytical methods (two nodes method and equivalent generator method) can be used when the VAC is presented by a formula.

Non-linear element is characterized by static and dynamic resistance. We can calculate them for every point of VAC (at fig.1.25 for work point – w.p.):

$$R_S = V_0 / I_0, \; R_д = \Delta V / \Delta I = dV / dI = tg\alpha,$$

α – the angle between axe X and tangent to working point (w.p.). $R_S > 0$, $R_д > 0$ when VAC rise and $R_д < 0$ when VAC drops.

Part 2. Alternating current (AC)

Instantaneous value of AC is a value at every time moment, so it depends on the time: $i(t) = I_m \sin(\omega t + \psi_I)$. Instantaneous value of alternating voltage is $v(t) = V_m \sin(\omega t + \psi_V)$ (fig.2.1).

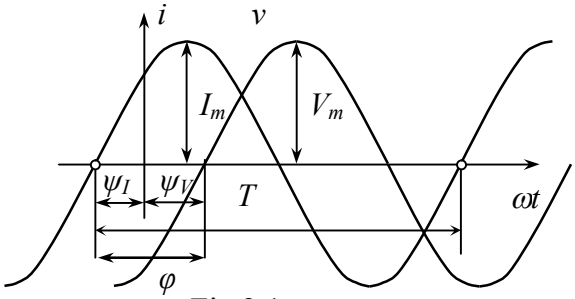

Fig.2.1

AC is characterized by such *parameters*: I_m - amplitude, maximum value during the period, period T, cyclic frequency $f = 1/T$ (quantity of periods per second) (Hz), angular frequency $\omega = 2\pi f$ (rad/s), phase $\beta = (\omega t + \psi_I)$, initial phase ψ_I (phase shift from zero).

Phase shift angle is: $\varphi = \psi_V - \psi_I$ (fig.2.1).

Average current value per half of period is*:*

$$I_{AV} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t \, dt \,,\ I_{AV} = 2I_m/\pi = 0.637 I_m .$$

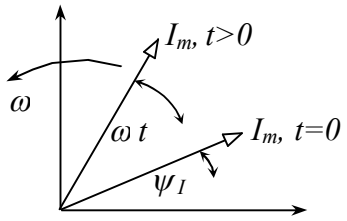

Fig.2.2

Effectiv e value of AC $i(t)$ (RMS – root-mean-square) is equal to such a value of DC I, which generates the same amount of energy per period $T = 2\pi$, as AC $i(t)$.

Amount of energy per period of AC:

$$Q_{\sim} = \int_0^T Ri^2 dt = RI_m^2 T \,.\int_0^T Ri^2 dt$$

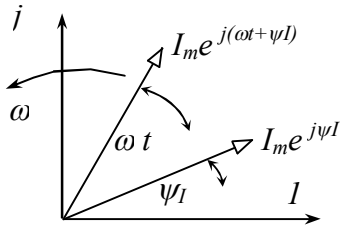

Fig.2.3

Amount of energy per half a period of DC: $Q_{-} = RI^2 T$.

$Q_{\sim} = Q_{-}$, so $Q_{\sim} = \int_0^T Ri^2 dt = RI^2 T$ and RMS value will be equal: $I = \sqrt{\frac{1}{T}\int_0^T Ri^2 dt}$.

$Q_{\sim} = RI_m^2 T = Q_{-} = RI^2 T$, thus AC effective value is $I = I_m/\sqrt{2} = 0.707 I_m$.

AC can be represented by the *time diagram* (fig.2.1), vector (fig.2.2) and complex number.

When AC $i = I_m \sin(\omega t + \psi_I)$ is represented by *vector*, the length of this vector is proportional to the amplitude I_m, and angle between this vector and axis X is $\beta = \omega t + \psi_I$. The positive rotation direction will be counterclockwise. In that case, the vectors of current and voltage will be rotating with the same angular frequency ω counterclockwise. It is convenient to fix them at the time moment $t = 0$ (fig.2.2), in that case the angle $\beta = \psi_I$ (initial phase).

Vector diagram consists of several vectors of currents and voltages, which represent real sinusoidal currents and voltages starting from the same point. It's better to build a vector diagram for the effective values of the currents and voltages $I = I_m/\sqrt{2}$, $V = V_m/\sqrt{2}$. One of the vectors is chosen as a basic one, it is the vector of current when the connection is in serial one and the vector of voltage when the

connection is in parallel.

The AC can also be designated by the *complex number* $i(t)=I_m\sin(\omega t+\psi_I)\div I_m e^{j(\omega t+\psi i)}=I_m e^{j\psi i}e^{j\omega t}=\underline{I}_m e^{j\omega t}$ (it's an exponential form of complex number). AC on the complex surface is shown on fig.2.3, where "+1" is a real axis and "j" is an imaginary axis. $\underline{I}_m=I_m e^{j\psi i}$ is then called an amplitude complex, $\underline{I}=(I_m/\sqrt{2})e^{j\psi i}$ is accordingly an effective complex that corresponds to the instantaneous current (at the moment $t=0$) and doesn't depend on the time (fig.2.3). Amplitude complex doesn't contain the frequency but it is not so important because circuit's voltage and current have the same frequency.

Complex numbers

Complex number $\underline{c}$ has two presentation forms: algebraic $\underline{c}=a+jb$ (where a is a real part and $b$is an imaginary part) and exponential $\underline{c}=ce^{j\alpha}$ (where c is a module and α is an argument) (fig.2.3a). One form can be converted into another by using the following expressions: $c=\sqrt{a^2+b^2}$, $\alpha = arctg\ (b/a)$, $a=c\cos\alpha$, $b=c\sin\alpha$, j is a symbol for the imaginary part (also known as rotating operator – see below why). Thus $\underline{c}=ce^{j\alpha}=a+jb$. It's more convenient to use the algebraic form when adding complex numbers $(a_1+jb_1)+(a_2+jb_2)=(a_1+a_2)+j(b_1+b_2)=a+jb$, and exponential form for multiplication and division of the complex numbers: $c_1e^{j\alpha_1}\cdot c_2e^{j\alpha_2}=c_1\cdot c_2e^{j(\alpha_1+\alpha_2)}=ce^{j\alpha}$, $\dfrac{c_1e^{j\alpha_1}}{c_2e^{j\alpha_2}}=\dfrac{c_1}{c_2}e^{j(\alpha_1-\alpha_2)}=ce^{j\alpha}$. The number $\underline{c}^*=a-jb=ce^{-j\alpha}$ is called a *complex conjugate* to number $\underline{c}=a+jb=ce^{j\alpha}$.

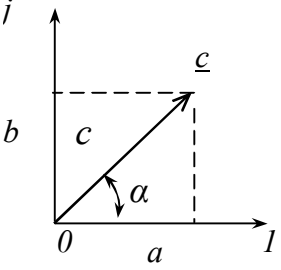

Fig.2.3a

Consumers at AC circuit

Expressions for instantaneous current and voltage are correspondingly:

$$i=I_m\sin(\omega t+\psi_I),\ v=V_m\sin(\omega t+\psi_U).$$

The voltage for the *resistive element* (fig.2.4) (active resistance) is

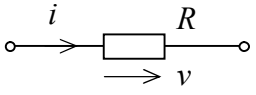

Fig.2.4

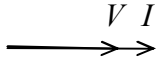

Fig.2.5

$$v=V_m\sin(\omega t+\psi_v)=Ri=RI_m\sin(\omega t+\psi_i)$$

according to Ohm`s law, where $V_m=RI_m$, $V=RI$, phase expression $\psi_v=\psi_i$ and phase shift angle makes $\varphi=\psi_v-\psi_i=0$. Resistance of this element is R (Ω) and conductance is thus $G=1/R$ (Sm). Vector diagram for this element is shown on fig.2.5. Active power of resistive element is accordingly $P=RI^2=GV^2$ (W).

Inductance L (H) is correspondingly the main parameter for the ideal *inductive element* (fig.2.6). The differential form of Ohm`s law is thus applied accordingly:

$$v_L=V_m\sin(\omega t+\psi_V)=L\frac{di_L}{dt}=L\frac{d}{dt}I_m\sin(\omega t+\psi_i)=$$

$$= \omega L I_m \cos(\omega t + \psi_i) = \omega L I_m \sin(\omega t + \psi_i + \pi/2),$$

where $V_m = \omega L I_m$, $V = X_L I$, reactance $X_L = \omega L (\Omega)$, susceptance $B_L = 1/\omega L$ (Sm), phase expression $\psi_v = \psi_i + \pi/2$, phase shift angle makes $\varphi = \psi_v - \psi_i = \pi/2$, it means voltage leads current. In case of DC: $\omega = 0$, $X_L = 0$, $B_L = \infty$. Vector diagram for this element is shown on fig.2.7. Reactive power for L element makes $Q_L = X_L I^2 = B_L V^2$ (VAr).

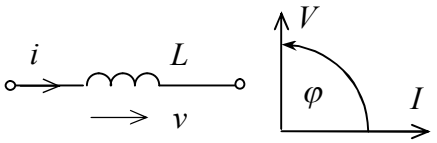

Fig.2.6

Fig.2.7

Capacitance C is the main parameter for the ideal *capacitive element* (fig.2.8). Integral form of Ohm`s law is applied in this case:

$$v_C = V_m \sin(\omega t + \psi_V) = \frac{1}{C}\int i\, dt = \frac{1}{C}\int I_m \sin(\omega t + \psi_i) = -\frac{1}{\omega C} I_m \cos(\omega t + \psi_i) =$$

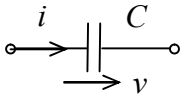

Fig.2.8

$$= -\frac{1}{\omega C} I_m \cos(\omega t + \psi_i) = \frac{1}{\omega C} I_m \sin(\omega t + \psi_i - \pi/2),$$

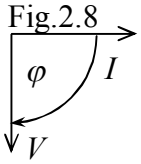

Fig.2.9

where $V_m = \frac{1}{\omega C} I_m$, $V = X_C I$, reactance $X_C = 1/(\omega C)$ (Ω), susceptance $B_C = \omega C$ (Sm) , phase expression $\psi_v = \psi_i - \pi/2$, phase shift angle makes $\varphi = \psi_v - \psi_i = -\pi/2$, it means voltage lags current. In case of DC : $\omega = 0$, $X_C = \infty$, $B_C = 0$. Vector diagram for this element is shown on fig.2.9. Reactive power for this element makes thus $Q_C = X_C I^2 = B_C V^2$ (VAr).

The complex designation for current, voltage, derivative and integral functions are accordingly:

$$i \div \underline{I} = \underline{I}_m / \sqrt{2} = I e^{j\psi_i},\ v \div \underline{V} = \underline{V}_m / \sqrt{2} = V e^{j\psi_V},$$

$$d/dt \div j\omega,\ \int dt \div 1/(j\omega) = -j/\omega\ .$$

Complex form of Ohm`s law equation for *R*-element is thus:

$$v = Ri \div \underline{V} = R\underline{I},\ V e^{j\psi_V} = RI e^{j\psi_i},$$

$$R = V e^{j\psi_V} / I e^{j\psi_i} = (V/I) e^{j(\psi_V - \psi_i)} = \mathrm{Re}^{j\varphi}, \varphi = 0.$$

Complex form of Ohm law equation for *L*-element is accordingly:

$$v = L di/dt \div \underline{V} = j\omega L \underline{I},\ V e^{j\psi_V} = j\omega L I e^{j\psi_i},$$

complex reactance is

$$j\omega L = V e^{j\psi_V} / I e^{j\psi_i} = (V/I) e^{j(\psi_V - \psi_i)} = X_L e^{j\varphi},\ \varphi = 90^\circ,$$

complex susceptance is $B_L e^{-j\varphi} = -j/\omega L$. Multiplication by j means counterclockwise rotation for $\varphi = 90^\circ$. That's why j is called a rotation operator.

Complex form of Ohm`s law equation for *C*-element is accordingly:

$$v = \int i dt / C \ \div\ \underline{V} = -j(1/\omega C)\underline{I},\ V e^{j\psi_V} = -j(1/\omega C) I e^{j\psi_i},$$

complex reactance is

$$-j(1/\omega C)=Ve^{j\psi_V}/Ie^{j\psi_i}=(V/I)e^{j(\psi_V-\psi_i)}=X_Ce^{j\varphi},\ \varphi=-90^\circ,$$

complex susceptance is $B_Ce^{-j\varphi}=j\omega C$. Multiplication by $-j$ means clockwise rotation for $\varphi=-90^\circ$.

Serial connection of consumers at AC circuit

Electrical status equations for the circuit (fig.2.10) for voltage instantaneous values and voltage vectors are accordingly:

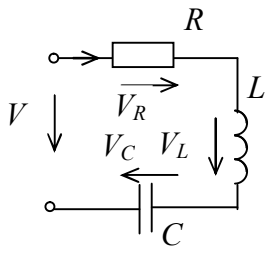

Fig.2.10

$$v_R+v_L+v_C=v,\ \bar{V}_R+\bar{V}_L+\bar{V}_C=\bar{V}.$$

Vector diagram is shown on fig 2.11. The calculated triangles for voltages, resistances and powers (fig.2.12) are obtained from this diagram. Out of those triangles:

$$V=\sqrt{V_R^2+(V_L-V_C)^2},\quad \varphi=arctg(V_L-V_C)/V_R,$$

$$V_R=V\cos\varphi=V_a,\ V_L-V_C=V\sin\varphi=V_r,$$

-these are active and reactive constituents of the applied voltage V. Therefore circuit impedance makes:

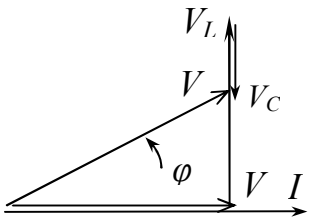

Fig.2.11

$$Z=\sqrt{R^2+(X_L-X_C)^2},\ \ \varphi=arctg((X_L-X_C)/R),$$

$R=Z\cos\varphi$, $X=X_L-X_C=Z\sin\varphi$ -

these are resistance and reactance of the circuit.

Total power makes thus:

$$S=\sqrt{P^2+(Q_L-Q_C)^2}\ (\text{VA}),\ \varphi=arctg(Q_L-Q_C)/P,$$

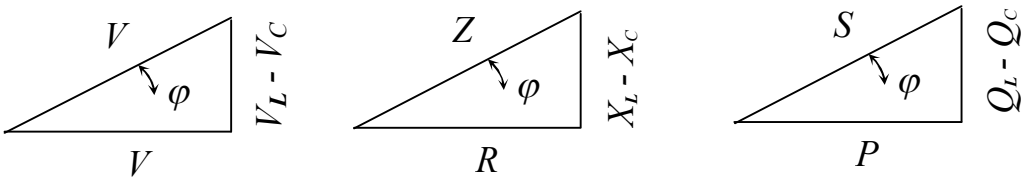

Fig.2.12

$P=S\cos\varphi=VI\cos\varphi$, $Q=Q_L-Q_C=S\sin\varphi=VI\sin\varphi$

- these are active and reactive powers of the circuit.

Circuit complex form electrical status equation is:

$$R\underline{I}+j\omega L\underline{I}-j/(\omega C)\underline{I}=\underline{V}.$$

Complex impedance makes thus: $\underline{Z}=R+j\omega L-j/(\omega C)=R+j(X_L-X_C)$.

Expression for Ohm`s law is accordingly:

$$\underline{Z}=\underline{V}/\underline{I}=\frac{Ve^{j\psi_V}}{Ie^{j\psi_I}}=\frac{V}{I}e^{j(\psi_V-\psi_I)}=Ze^{j\varphi}=Z\cos\varphi-jZ\sin\varphi=R+jX.$$

Parallel connection of consumers at AC circuit

Circuit electrical status equations (fig.2.13) for current instantaneous values and current vectors are accordingly:

$$i_R+i_L+i_C=i,\ \bar{I}_R+\bar{I}_C+\bar{I}_L=\bar{I}.$$

Vector diagram is shown on fig.2.14. The calculated triangles of currents and conductivities are obtained from this diagram (fig.2.15). From those triangles we get subsequently:

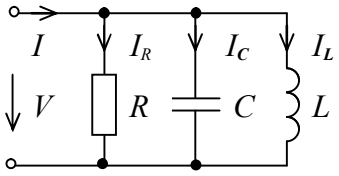

Fig.2.13

$$I = \sqrt{I_R^2 + (I_C - I_L)^2}, \qquad \varphi = arctg((I_C - I_L)/I_R),$$

$I_R = I\cos\varphi = I_a$, $I_C - I_L = I\sin\varphi = I_r$ - these are active and reactive constituents of the current.

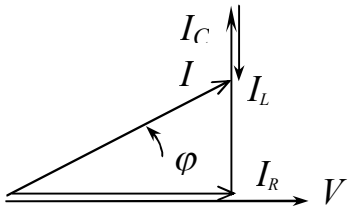

Fig.2.14

Circuit admittance makes:

$$Y = \sqrt{G^2 + (B_C - B_L)^2}, \qquad \varphi = arctg(B_C - B_L)/G,$$

$$G = Y\cos\varphi,\ B = B_C - B_L = Y\sin\varphi$$

- these are conductance and susceptance of the circuit.

Electrical status equation for the circuit in complex form is accordingly: $G\underline{V} + j\omega C\underline{V} - j/(\omega L)\underline{V} = \underline{I}$.

Complex admittance makes: $\underline{Y} = G + j\omega C - j/(\omega L) = G + j(B_C - B_L)$.

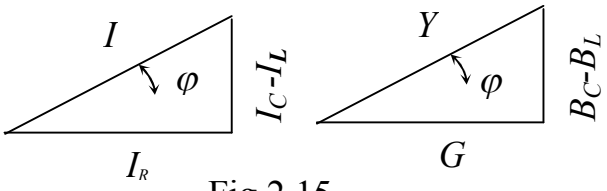

Fig.2.15

Expression for Ohm`s law is thus: $\underline{Y} = \underline{I}/\underline{V} = \frac{Ie^{j\psi_V}}{Ve^{j\psi_I}} = \frac{I}{V}e^{j(\psi_I - \psi_V)} =$

$$Ye^{-j\varphi} = Y\cos\varphi - jY\sin\varphi = G + jB.$$

Total complex power makes accordingly:

$$\underline{S} = \underline{V}\underline{I}^* = Ve^{j\psi_V} \cdot Ie^{-j\psi_I} = VIe^{j(\psi_V - \psi_I)} = Se^{j\varphi} =$$
$$= S\cos\varphi + jS\sin\varphi = P + jQ,$$

where real part of complex number $P = S\cos\varphi$ – is an active power, imaginary part of complex number $Q = S\sin\varphi$ – is a reactive power.

To check the calculation of the circuit you may use *power balance equations*: the active power of the source must be equal to the active powers of the consumers:

$$P_{ps} = \sum P_{cons},$$

$$P_{ps} = VI\cos\varphi,\ \sum P_{cons} = R_1I_1^2 + R_2I_2^2 + ... = \sum R_nI_n^2,$$

the reactive power of the source must be equal to the reactive powers of the consumers: $Q_{ps} = \sum Q_{cons}$,

$$Q_{ps} = VI\sin\varphi,\ \sum Q_{cons} = X_1I_1^2 + X_2I_2^2 + ... = \sum X_nI_n^2,$$

where I_n – is an effective value of the branch n-th current, R_n– resistance of the n-th branch, $X_n = X_{Ln} - X_{Cn}$ – reactance of the n-th branch.

The transformation formulas must be used to calculate the alternating current

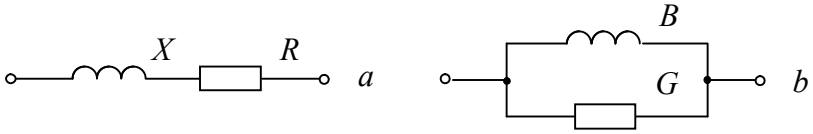

Fig.2.16

circuits. The admittance is inversely to impedance:

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = \frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2} = G - jB\ .$$

So, the following formulas must be used to transform serial connection into parallel (fig.2.16):

$$G = \frac{R}{(R^2 + X^2)},\ B = -\frac{X}{(R^2 + X^2)}.$$

It is obtained from the calculation triangles of resistances (Fig.2.17a) and conductivities (Fig.2.17b):

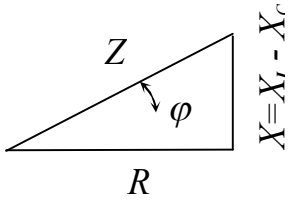

Fig.2.17a

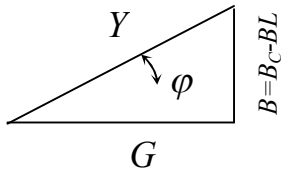

Fig.2.17b

$$\cos\varphi = \frac{R}{Z} = \frac{G}{Y},\ \sin\varphi = \frac{X}{Z} = \frac{B}{Y},$$

Thus the following formulas must be used to transform parallel connection into serial (fig.2.18):

$$R = \frac{GZ}{Y} = \frac{G}{Y^2} = \frac{G}{G^2 + B^2}\ ,\ X = \frac{BZ}{Y} = \frac{B}{Y^2} = \frac{B}{G^2 + B^2}.$$

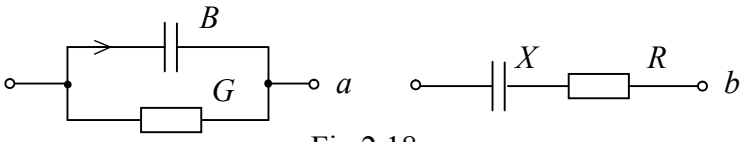

Fig.2.18

The real coil can be represented by serial and parallel substitution schemes (fig.2.16). The elements of this scheme: L - is an ideal inductance $X = \omega L,\ B = 1/(\omega L)$, $R\,(G)$ – represents power losses. The coil quality can be estimated by Q - *factor*:

$$d = Q/P = X_L/R = tg\varphi.$$

The real capacitor can be represented by serial and parallel substitution schemes (fig.2.18). The elements of this scheme: C - is an ideal capacitance $B = \omega C,\ X = 1/(\omega C)$ $R\,(G)$ – represents power losses.

The capacitor quality can be estimated by *loss tangent*:

$$tg\delta = P/Q = R/X_C,\ \delta = 90^\circ - \varphi.$$

Power factor determines the efficiency of using electrical energy:

$$\cos\varphi = P/S = P/\sqrt{P^2 + Q^2}\ ,$$

P - is an active useful power, Q - reactive, non-useful power (for electromagnetic field creation).

In ideal case power factor depends on the loading character

$$\cos\varphi = R/Z = R/\sqrt{R^2 + X^2}\ .$$

$\cos\varphi = 1$ must be provided to avoid the work of electrical devices at idle mode. Most of the devices consume the active-inductive power (P and Q_L). Capacitors C must be connected in parallel to such devices to enhance $\cos\varphi$, thus $Q_L = Q_C$. Reactive power, which is non-useful power $Q_L - Q_C = Q = 0$, so $\cos\varphi = 1$ is maximum.

Voltage resonance.

Voltage resonance take place at the circuit with serial connection of *L,C* elements (fig.2.10). $V_L = V_C$ at resonance mode, so the condition of voltage resonance is $X_L = X_C$, it means

$$\omega_0 L = 1/(\omega_0 C),$$

Thus $\omega_0^2 LC = 1$ and, resonance frequency

$$\omega_0 = 1/\sqrt{LC}\,.$$

Resonance can be reached by changing C, L or ω_0.

$\rho = \omega_0 L = 1/(\omega_0 C)$, $\rho = \sqrt{L/C}$ is called wave resistance.

At resonance mode

$$X = X_L - X_C = 0,\ Z = \sqrt{R^2 + X^2} = R,$$

$$Q_L - Q_C = Q = 0,\ S = \sqrt{P^2 + Q^2} = P,\ \varphi = 0\,.$$

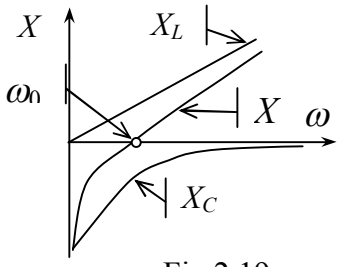

Fig.2.19

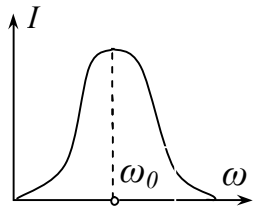

Fig.2.20

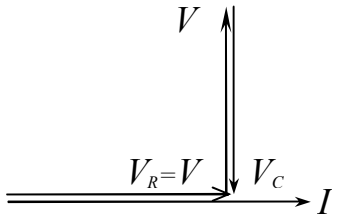

Fig.2.21

Total current $I = V/Z = V/R$ is at maximum, what is an indication of the voltage resonance. Frequency characteristics of the circuit $X_L(\omega) = \omega L$, $X_C(\omega) = 1/(\omega C)$, $X(\omega) = X_L(\omega) - X_C(\omega)$ are shown at fig.2.19. When $\omega < \omega_0$, $X < 0$, $\varphi < 0$, reactance has inductive character. When $\omega > \omega_0$ $X > 0$, $\varphi > 0$ reactance has capacitive character.

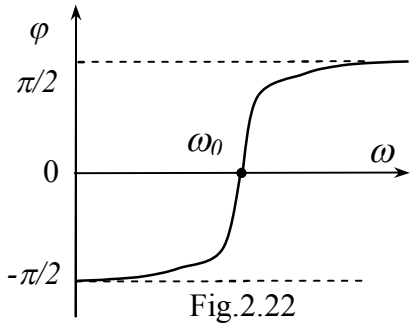

Fig.2.22

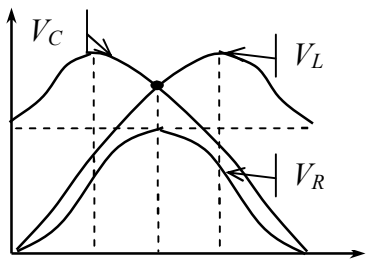

Fig.2.23

At fig.2.20 resonance curve $I(\omega)$ and at fig.2.21 vector diagram for resonance

mode are shown.

Phase-frequency characteristic $\varphi(\omega) = arctg\dfrac{\omega L - 1/(\omega C)}{R}$ is shown at fig.2.22 and the resonance curves of voltages $V_R(\omega)$, $V_L(\omega)$, $V_C(\omega)$ at fig.2.23 accordingly.

Voltage resonance should be avoided, because the voltage across the elements may several times exceed the nominal value.

Current resonance.

Current resonance takes place at the circuit with parallel connection of *L,C* elements (fig.2.24). $I_L = I_C$ at resonance mode, so the condition of voltage resonance for real circuit is $B_L = B_C$, that means

$$\omega_0 L/(R^2 + (\omega_0 L)^2) = 1/(\omega_0 C).$$

For ideal circuit ($R = 0$) the condition is $\omega_0 L = 1/(\omega_0 C)$.

Thus $\omega_0^2 LC = 1$, resonance frequency $\omega_0 = 1/\sqrt{LC}$.

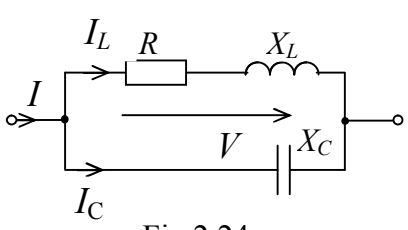

Fig.2.24

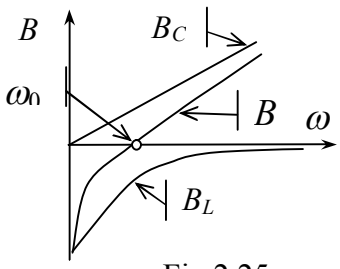

Fig.2.25

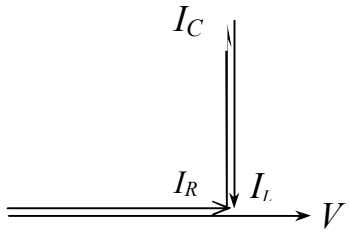

Fig.2.26

The resonance can be reached by changing C, L or ω_0.

At resonance mode

$$B = B_C - B_L = 0,\ Y = \sqrt{G^2 + B^2} = G,$$

$$Q_L - Q_C = Q = 0,\ S = \sqrt{P^2 + Q^2} = P,\ \varphi = 0.$$

Total current $I = VY = VG$ is at minimum, what is the indication of the current

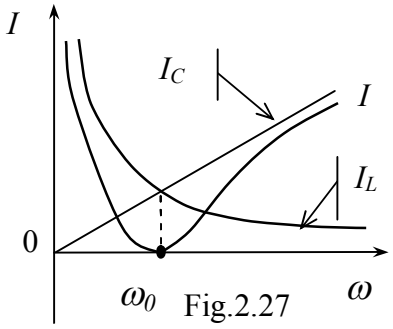

Fig.2.27

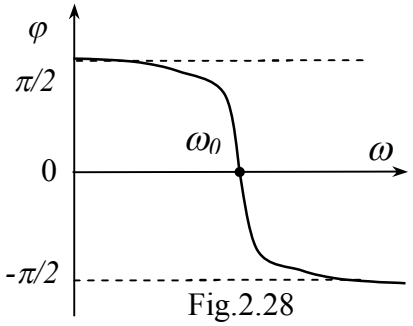

Fig.2.28

resonance.

Frequency characteristics of the ideal ($R=0$) circuit $B_L(\omega) = 1/(\omega L)$, $B_C(\omega) = \omega C$, $B(\omega) = B_C(\omega) - B_L(\omega)$ are shown at fig.2.25. Susceptance has an inductive character when $\omega < \omega_0$, $B < 0$, $\varphi < 0$. Susceptance has a capacitive

character, when $\omega > \omega_0$ $B > 0$, $\varphi > 0$.

Vector diagram for resonance mode is shown at fig.2.26. Resonance curves $I(\omega)$, $I_L(\omega) = B_L V$, $I_C(\omega) = B_C V$ and phase-frequency characteristic $\varphi(\omega)$ are shown at fig.2.27 and 2.28.

Voltage resonance on one hand should be avoided, because the current across the elements may several times exceed the nominal current, but on the other hand the resonance can be applied for rising power factor and as the working mode of some electronic devices.

Part 3. Three-phase circuits

Three-phase electro-motive-force circuit system is the set of three sinusoidal e.m.f. with the same frequency ω and out of phase with each other by $2\pi/3$ (120°). *Phase* is the part of the circuit with the same current. The amplitudes of e.m.f. are marked accordingly: E_{Am}, E_{Bm}, E_{Cm}, if they are equal, such system is called *balanced*.

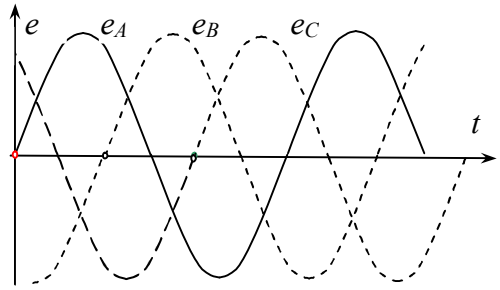

Fig.3.1

The instantaneous values of e.m.f. (fig.3.1) are:

$$e_A = E_{Am}\sin\omega t,\ e_B = E_{Bm}\sin(\omega t - 120^\circ),\ e_C = E_{Cm}\sin(\omega t + 120^\circ).$$

Phase sequence is the time order in which the e.m.f. pass through their respective maximum values (or through zero value). Phase sequence *ABC* is called positive (fig.3.1), the reverse phase sequence *ACB* be called negative.

The following requirements are met for three-phase balanced electro-motive force system:

$$E_A = E_B = E_C = E_{ph}.$$

The following expressions are true having disregarded losses at power sources:

$$E_A = V_A, \qquad E_B = V_B, \qquad E_C = V_C,$$

where V_A, V_B, V_C – are *source phase voltages* (between the lines and neutral point *N* (fig.3.3). These voltages in complex form are presented as:

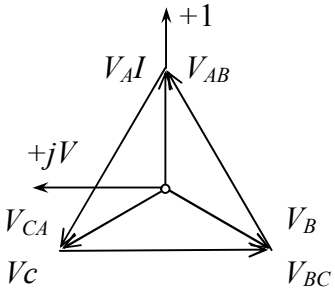

Fig.3.2

$$\underline{V}_A = V_A e^{j0}, \qquad \underline{V}_B = V_B e^{-j120^\circ}, \qquad \underline{V}_C = V_C e^{j120^\circ}.$$

Linear voltages (between lines, which connect the sources and the consumers) (fig.3.4) in complex form are:

$$\underline{V}_{AB} = \underline{V}_A - \underline{V}_B = V_{AB} e^{j30^\circ},$$

$$\underline{V}_{BC} = \underline{V}_B - \underline{V}_C = V_{BC} e^{-j90^\circ}$$

$$\underline{V}_{CA} = \underline{V}_C - \underline{V}_A = V_{CA} e^{j150^\circ}.$$

Linear voltage is equal to the difference between corresponding phase voltages and lead the phase of the first one for 30° (fig.3.2). Vector diagram (fig.3.2) illustrates relationship between phase and linear voltages.

Three-phase circuit consists of three-phase electro-motive force system, three-phase loads and connection wires.

The most common types of connection the three-phase sources and consumers are WYE (Y) (fig.3.3) and DELTA (Δ) (fig.3.7).

At WYE connection the ends of source phases windings (fig.3.3) are connected in common neutral point N, and the beginnings of phases A,B,C are connected to the linear wires. The ends of consumer phase windings (fig.3.3) are connected in common neutral point n, and the beginnings of phases a,b,c are connected to the linear wires.

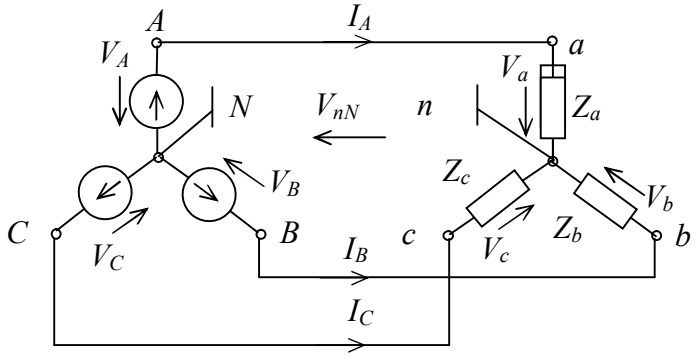

Fig.3.3

The source phase voltages are called the voltages between phase and neutral points V_A,V_B,V_C, for consumer V_a,V_b,V_c. The source linear voltages are called the voltages between phase points (fig.3.4) V_{AB},V_{BC},V_{CA}, for consumer V_{ab},V_{bc},V_{ca}. The directions of these voltages are shown at fig.3.4. The effective values of phase and linear voltages are related according to the expression $V_L = \sqrt{3}V_{ph}$.

For WYE connection (fig.3.4) *phase currents* (flowing through the phase) I_{ph}

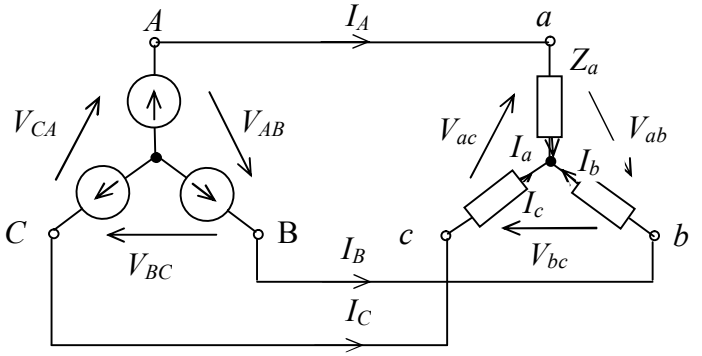

Fig.3.4

(I_a,I_b,I_c), are equal to the *linear currents* (flowing through the lines connecting the source and the consumer) I_L (I_A,I_B,I_C), $I_{ph} = I_L$. The directions of these currents are shown at fig.3.4. Balanced load is one in which the phase impedances are equal in magnitude and in phase:

$$\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = \underline{Z}_{ph}.$$

In this case:

$$V_a = V_A, \qquad V_b = V_B, \qquad V_c = V_C.$$

$$\underline{I}_A = \underline{V}_a / \underline{Z}_a, \qquad \underline{I}_B = \underline{V}_b / \underline{Z}_b, \qquad \underline{I}_C = \underline{V}_c / \underline{Z}_c.$$

The effective values of the currents are also equal: $I_A = I_B = I_C = I_{ph} = I_L$.

If the load is unbalanced ($\underline{Z}_a \neq \underline{Z}_b \neq \underline{Z}_c$) the voltage between the neutral points of source and consumer appears – $\underline{V}_{nN}$ (fig.3.3). This voltage is called the *bias*

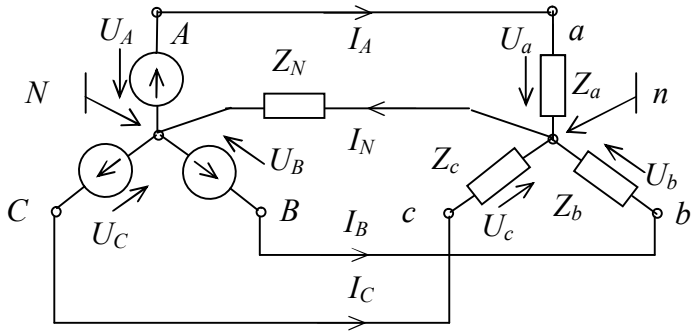

Fig.3.5

neutral and can be calculated by using the method of two nodes:

$$\underline{V}_{nN} = \frac{\underline{V}_A \underline{Y}_a + \underline{V}_B \underline{Y}_b + \underline{V}_C \underline{Y}_c}{\underline{Y}_a + \underline{Y}_b + \underline{Y}_C},$$

where $\underline{Y}_a = 1/\underline{Z}_a = I_a / V_a, \qquad \underline{Y}_b = 1/\underline{Z}_b = I_b / V_b, \qquad \underline{Y}_c = 1/\underline{Z}_c = I_c / V_c.$

In that case the consumer phase voltages are calculated according to the following expressions:

$$\underline{V}_a = \underline{V}_A - \underline{V}_{nN}, \quad \underline{V}_b = \underline{V}_B - \underline{V}_{nN}, \quad \underline{V}_c = \underline{V}_C - \underline{V}_{nN},$$

Phase currents complexes are:

$$\underline{I}_a = \underline{V}_a / \underline{Z}_a, \; \underline{I}_b = \underline{V}_b / \underline{Z}_b, \; \underline{I}_c = \underline{V}_c / \underline{Z}_c.$$

There is also a *neutral wire* at three-phase four-wires circuits, which connects neutral points of source N and consumer n (fig.3.5). In this case $\underline{V}_{nN} = 0$.

The following is true according to the Kirchhoff`s first law for node n:

$$\underline{I}_A + \underline{I}_B + \underline{I}_C = \underline{I}_N.$$

When the load is balanced ($\underline{Z}_a = \underline{Z}_b = \underline{Z}_c$): $\underline{I}_A + \underline{I}_B + \underline{I}_C = 0, \; I_N = 0, \; \underline{V}_{nN} = 0$.

The vector diagram of currents for unbalanced load is shown in fig.3.6.

At DELTA connection the end of one source (consumer) winding is connected to the beginning to the second source (consumer) winding (fig.3.7). For this

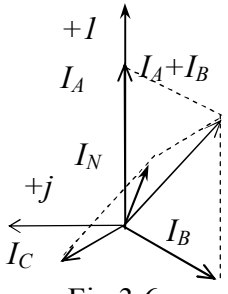

Fig.3.6

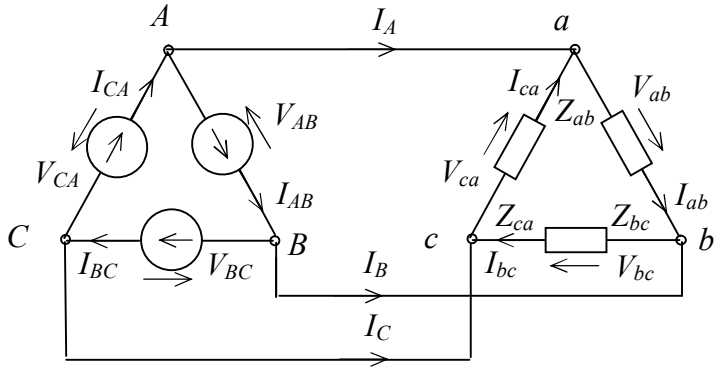

Fig.3.7

connection the following is true: $V_{ph}=V_L$, $V_{AB}=V_{BC}=V_{CA}=V_L$.

The phase (linear) complex voltages can be represented:

$$\underline{V}_{AB}=V_{AB}e^{j0}, \qquad \underline{V}_{BC}=V_{BC}e^{-j120^\circ}, \qquad \underline{V}_{CA}=V_{CA}e^{j120^\circ}.$$

The consumer linear (phase) voltages are equal to the source linear voltages:

$$\underline{V}_{ab}=\underline{V}_{AB}, \qquad \underline{V}_{bc}=\underline{V}_{BC}, \qquad \underline{V}_{ca}=\underline{V}_{CA}.$$

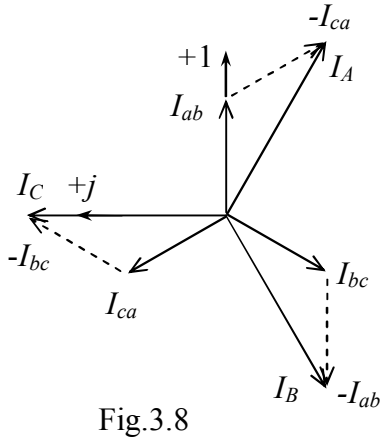

Fig.3.8

If the phase load is active ($\varphi = 0$), the vectors of phase currents I_{ab}, I_{bc}, I_{ca} have the same directions as the vectors of corresponding phase voltages V_{AB}, V_{BC}, V_{CA}.

If the phase load is an active-inductive one ($\varphi > 0$), the phase current lags behind the corresponding phase voltage by an angle of $\varphi = arctg(X_{ph}/R_{ph})$.

If the phase load is an active-capacitive one ($\varphi < 0$), the phase current leads the corresponding phase voltage by an angle of $\varphi = arctg(X_{ph}/R_{ph})$.

The load is balanced when $\underline{Z}_{ab}=\underline{Z}_{bc}=\underline{Z}_{ca}=\underline{Z}_{ph}$ and unbalanced when $\underline{Z}_{ab}\neq\underline{Z}_{bc}\neq\underline{Z}_{ca}$.

The following is true for the nodes a,b,c (fig.3.7) according to the first Kirchhoff law:

$$\underline{I}_A+\underline{I}_{ca}-\underline{I}_{ab}=0, \qquad \underline{I}_B+\underline{I}_{ab}-\underline{I}_{bc}=0, \qquad \underline{I}_C+\underline{I}_{bc}-\underline{I}_{ca}=0,$$

Then:

$$\underline{I}_A=\underline{I}_{ab}-\underline{I}_{ca}, \qquad \underline{I}_B=\underline{I}_{bc}-\underline{I}_{ab}, \qquad \underline{I}_C=\underline{I}_{ca}-\underline{I}_{bc}.$$

The linear current is equal to the difference between corresponding phase currents and lags the first one for 30° (fig3.8). Vector diagram (fig.3.8) illustrates relationship between phase and linear currents.

The effective values of the phase and the linear currents are connected by expression: $I_L=\sqrt{3}\,I_{ph}$.

Complex phase currents can be defined according to Ohm's law:

$$\underline{I}_{ab}=\underline{V}_{ab}/\underline{Z}_{ab}, \ \underline{I}_{bc}=\underline{V}_{bc}/\underline{Z}_{bc}, \ \underline{I}_{ca}=\underline{V}_{ca}/\underline{Z}_{ca}.$$

For balanced load: $I_A=I_B=I_C$, $I_{ab}=I_{bc}=I_{ca}$.

Complex total power of three-phase unbalanced circuit is:

$$\underline{S}=\underline{V}_A\underline{I}_A^*+\underline{V}_B\underline{I}_B^*+\underline{V}_C\underline{I}_C^*=P+jQ.$$

Active power of three-phase unbalanced circuit is:

$$P=V_AI_A\cos\varphi_A+V_BI_B\cos\varphi_B+V_CI_C\cos\varphi_C=P_A+P_B+P_C.$$

Reactive power of three-phase unbalanced circuit is:

$$Q=V_AI_A\sin\varphi_A+V_BI_B\sin\varphi_B+V_CI_C\sin\varphi_C=Q_A+Q_B+Q_C.$$

These formulas can be used for WYE or DELTA connections.

Active P, reactive Q and total S powers of the consumer can be calculated by using phase or linear voltages for balanced load:

$$P = P_a + P_b + P_c = 3P_{ph} = 3V_{ph} I_{ph} \cos\varphi_{ph} = \sqrt{3} V_L I_L \cos\varphi_L ,$$

$$Q = Q_a + Q_b + Q_c = 3Q_{ph} = 3V_{ph} I_{ph} \sin\varphi_{ph} = \sqrt{3} V_L I_L \sin\varphi_L ,$$

$$\cos\varphi_{ph} = R_{ph} / Z_{ph} , \qquad \sin\varphi_{ph} = X_{ph} / Z_{ph} ,.$$

$$S = S_a + S_b + S_c = 3S_{ph} = 3V_{ph} I_{ph} , \qquad S = \sqrt{P^2 + Q^2} .$$

The same formulas can be used for WYE and DELTA connection.

Part 4. The non-sinusoidal current circuits.

Non-sinusoidal voltages or currents are the ones which are changed with the time according to periodical non-sinusoidal law. The cause of non-sine currents (voltages) is the source of non- sinusoidal voltage or the non-linear element of the circuit.

Such circuits may be represented by the Fourier series as the sum of sinusoidal functions in order to get calculated:

$$v = V_0 + V_{m1} \sin(\omega t + \psi_{V1}) + V_{m2} \sin(2\omega t + \psi_{V2}) + \ldots + V_{mk} \sin(k\omega t + \psi_{Vk}) =$$

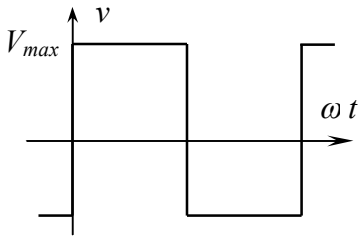

Fig.4.1

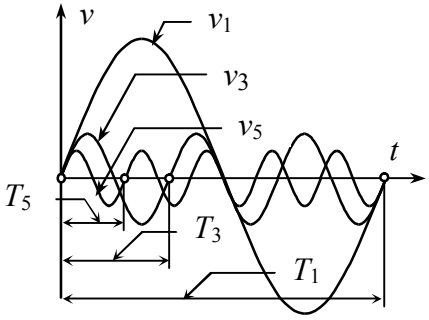

Fig.4.2

$$= V_0 + \sum_{k=1}^{\infty} V_{mk} \sin(k\omega t + \psi_{Vk}) ,$$

where V_0 is the steady component; $v_1 = V_{m1} \sin(\omega t + \psi_{V1})$ is the first (basic) harmonic component, (ω - the frequency of first harmonic), $v_k = V_{mk} \sin(k\omega t + \psi_{Vk})$ - k harmonic component (called also as *harmonic*), V_{mk} - amplitude, ω - fundamental frequency, $k\omega$ - frequency of k harmonic, ψ_{Vk} - initial phase of k harmonic. The harmonics with the frequencies 2, 3,… k times larger than ω, are called higher harmonics.

We can represent the value $V_{mk} \sin(k\omega t + \psi_{Vk}) \div A_{mk} \sin(k\omega t + \varphi_k)$ by the sum of two constituents :

$$A_{mk} \sin(k\omega t + \varphi_k) = B_{mk} \sin k\omega t + C_{mk} \cos k\omega t ,$$

where $B_{mk} = A_{mk} \cos\varphi_k$, $C_{mk} = A_{mk} \sin\varphi_k$, $A_{mk} = \sqrt{B_{mk}^2 + C_{mk}^2}$, $\varphi_k = arctg(C_{mk} / B_{mk})$.

So, the Fourier series we can write down ($v \div f(\omega t)$):

$$f(\omega t) = A_0 + \sum_{k=1}^{\infty} B_{mk} \sin k\omega t + \sum_{k=1}^{\infty} C_{mk} \cos k\omega t .$$

If the function is symmetrical across the X axis $f(\omega t) = -f(\omega t \pm \pi)$ then

Fourier series have only odd harmonics:

$$f(\omega t) = A_{m1}\sin(\omega t + \varphi_1) + A_{m3}\sin(3\omega t + \varphi_3) + A_{m5}\sin(5\omega t + \varphi_5) + ... =$$
$$= B_{m1}\sin\omega t + C_{m1}\cos\omega t + B_{m3}\sin 3\omega t + C_{m3}\cos 3\omega t + B_{m5}\sin 5\omega t + C_{m5}\cos 5\omega t + ...$$

If the function is symmetrical across the origin $f(\omega t) = -f(-\omega t)$ then Fourier series have only sin constituents:

$$f(\omega t) = B_{m1}\sin\omega t + B_{m2}\sin 2\omega t + B_{m3}\sin 3\omega t + ...$$

If the function is symmetrical across Y axis $f(\omega t) = f(-\omega t)$ then Fourier series have only steady component and cos constituents:

$$f(\omega t) = A_0 + C_{m1}\cos\omega t + C_{m2}\cos 2\omega t + C_{m3}\cos 3\omega t + ...$$

Fourier series has only steady component and cos constituents:

$$f(\omega t) = A_0 + C_{m1}\cos\omega t + C_{m2}\cos 2\omega t + C_{m3}\cos 3\omega t + ...$$

For example, the square shape of voltage (fig.4.1) can be represented in such a way (fig.4.2):

$$v = \frac{4V_{\max}}{\pi}(\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t),$$

Non-sinusoidal current $i = I_0 + \sum_{k=1}^{\infty} I_{mk}\sin(k\omega t + \psi_{Ik})$ (i.e. the sum of the sinusoidal currents) is present in the circuit with non-sinusoidal voltage $v = V_0 + \sum_{k=1}^{\infty} V_{mk}\sin(k\omega t + \psi_{Vk})$ (the sum of the sine voltages). The calculation of the circuit is based on the principle of superposition. The steady component of the current I_0 can be calculated by using the methods of DC circuits' calculation and harmonic of current i_k by using the methods of AC circuits' calculation.

As known reactance of the coil for *k*-harmonic is equal $X_{Lk} = k\omega L = kX_L$ and susceptance $B_{Lk} = 1/(k\omega L) = B_L/k$. Reactance of the coil for DC (as effect of the steady voltage component V_0) is $X_L(0) = 0 \cdot L = 0$. The susceptance of the capacitor for *k*-harmonic is $B_{Ck} = k\omega C = kB_C$ and reactance is $X_{Ck} = 1/(k\omega C) = X_C/k$. Reactance of the capacitor for DC (as effect of the steady voltage component V_0) is $X_C(0) = 1/(0 \cdot C) = \infty$, $I_0 = 0$. The resistance of the circuit doesn't actually depend on the frequency and is the same for every harmonic.

The non-sinusoidal circuit calculation order is:

– the source voltage is expressed by Fourier series as an infinite sum of harmonic (sinusoidal) components (functions);

– the circuit for every harmonic component is calculated separately using DC and AC circuits' calculation methods. Also it should be taken into consideration that the reactances depend on the frequency;

– according to the superposition principle, the current instantaneous value is equal to the sum of currents instantaneous values of all harmonics, that's why the calculation results are considered at each particular moment. The effective values of voltage and current are equal correspondingly:

$$V = \sqrt{V_0^2 + V_1^2 + ... + V_k^2}\ ,\ I = \sqrt{I_0^2 + I_1^2 + ... + I_k^2}\ ,$$

where V_k, I_k are harmonic voltages and currents effective values.

The *average value* of non-sinusoidal function A_0 (V_0, I_0) for the period:

$$A_0 = A_{AV} = \frac{1}{T}\int_0^T a\,dt\,,$$

The *effective value* of non-sinusoidal function - *A (V, I)* is the mean-square value for the period T :

$$A = \sqrt{\frac{1}{T}\int_0^T a^2 dt} = \sqrt{\sum_{k=o}^{n} A_k^2} = \sqrt{A_0^2 + ... + A_k^2}\ .$$

Shape factor is equal to the relation of function effective value to its average value: $K_{sh} = A / A_{AV}$. ($K_{sh} = 2/\pi = 1.11$ for sinusoidal curve).

Amplitude factor is equal to the relation of function amplitude value to its effective value: $K_a = A_m / A$. ($K_a = \sqrt{2} = 1.41$ for sinusoidal curve).

Distortion factor is equal the relation of first harmonic effective value to the function effective value:

$$K_d = A_1 / A \ (\ K_d = 1 \text{ for sinusoidal curve}).$$

Harmonic factor is equal the relation of high harmonics effective values to the first harmonic effective value: $K_g = A_g / A_1$, where $A_g = \sqrt{A_2^2 + ... + A_k^2} = \sum_{k=2}^{\infty} A_k^2$ is the mean-square value of high harmonics effective values ($K_g = 0$ for sinusoidal curve).

Active power of non- sinusoidal current is equal to the sum of harmonics active powers: $P = \sum_{k=o}^{n} V_k I_k = P_0 + P_1 + .. + P_k = P_0 + \Sigma P_k$, where $P_0 = V_0 I_0$ is the power of steady voltage component, $P_k = V_k I_k \cos\varphi_k$ is the active power of k harmonic, $\varphi_k = \psi_{Vk} - \psi_{Ik}$ - phase shift angle between k harmonic component of voltage and current. Reactive power of non- sinusoidal current is equal to the sum of harmonics reactive powers: $Q = \Sigma V_k I_k \sin\varphi_k = \Sigma Q_k$.

Total power of non- sinusoidal current is: $S = \sqrt{P^2 + Q^2}$.

Part 5. Transient processes

The transient processes occur when devices and circuits change their working regime. Transient processes may have negative effect in electrical engineering, but they can be useful in electronics.

The transient processes start at turning on/off the sources, changing the configuration of the scheme, circuit parameters, changing the current/voltage amplitude, phase, frequency or shape. Still the transient processes are typically caused by commutation (turning on/off the circuit).

The transient process is the process of transition from one energetic state of the circuit into another. This process cannot proceed stepwise, because the stock of energy can´t change abruptly. That's because the elements' values upon which the

energy storage depends (*L,C*) don't allow to change current and voltage stepwise (i_L , v_C). Two main laws of transient processes come out from this point.

The first law states that the current through inductance just after the commutation $i_L(0+)$ is equal to the current through inductance just before the commutation $i_L(0–)$: $i_L(0+)=i_L(0–)=i_L(0)$.

The second law states that the voltage at capacity just after the commutation $v_C(0+)$ is equal to the voltage at capacity just before the commutation $v_C(0–)$: $v_C(0+)=v_C(0–)=v_C(0)$.

Initial conditions (voltage or current values at the commutation moment $t=0$) are defined by these laws. The steady-state mode before the commutation is at $t<0$. The steady-state mode after the commutation is after the transient process is over.

The transient process duration depends on the elements parameters. It is estimated as $t_{tr}=5\div 6\tau$, where τ is the time constant. It is time during which voltage or current changes *e*=2.7 times of its initial value.

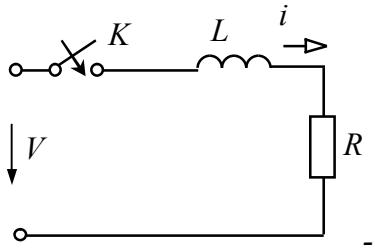

Fig.5.1

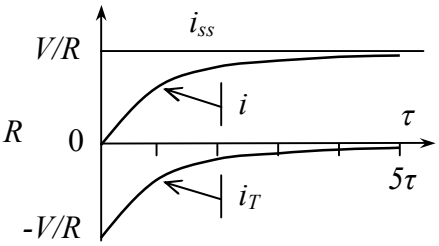

Fig.5.2

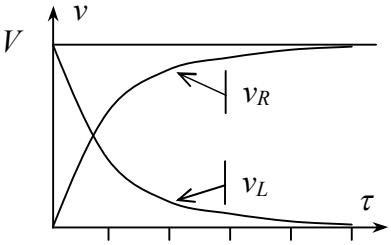

Fig.5.3

The transient process can be described by linear differential equation, which is formed with the help of Kirchhoff's laws. Commutation laws should be used to solve this equation.

The partial solution of inhomogeneous differential equation is the steady-state component i_{SS} or v_{SS} . The general solution of homogeneous differential equation is the transient component i_T or u_T, which dies out with time. The solution of linear differential equation is current (voltage), which is equal to the sum of transient and steady-state components $i(t)=i_T+i_{SS}$ ($v(t)=v_T+v_{SS}$). Therefore, to calculate transient process means to find the current or voltage changing rule.

Let's analyse the transient process when *RL* link is connected to DC source (fig.5.1). According the differential equation to the Kirchhoff's second law for after commutation steady-state mode is: $Ldi/dt+Ri=V$. Its solution is $i(t)=i_T+i_{SS}$. The partial solution i_{SS} of inhomogeneous differential equation $Ldi_{SS}/dt+Ri_{SS}=V$ is equal to the current value when transient process is over $i_{SS}=V/R$ (because $X_L=0$ for DC).

i_T is the general solution of homogeneous differential equation $Ldi_T/dt+Ri_T=0$. The characteristic equation corresponding to this differential one is $pL+R=0$ with its root $p=-R/L$. The time constant is $\tau=1/p=L/R$. Since the

characteristic equation has one real root, the transient component is $i_T = A e^{pt}$. Constant of integration can be found from initial conditions: $i(0) = i_T(0) + i_{SS}(0) = A + V/R$. According to the first commutation law $i(0) = i(0–) = 0$, so $A = -V/R$, $i_T = -V/R\, e^{-(R/L)t}$.

The solution of differential equation is (fig.5.2):

$$i = i_T + i_{SS} = -V/R\, e^{-(R/L)t} + V/R = V/R(1 - e^{-(R/L)t}).$$

The voltage on resistive element is (fig.5.3):

$$v_R = Ri = R(V/R)(1 - e^{-(R/L)t}) = V(1 - e^{-(R/L)t}).$$

The voltage on inductive element is (fig.5.3):

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}\left(\frac{V}{R}(1 - e^{-(R/L)t})\right) = L\frac{V}{R}(-\frac{R}{L})\cdot(-e^{-(R/L)t}) = Ve^{-(R/L)t}.$$

Let's analyse the transient process when *RL* link is disconnected from DC source and shortened (fig.5.4). The differential equation according to the Kirchhoff's second law for after commutation steady-state mode is: $Ldi/dt + Ri = 0$. Its solution is $i(t) = i_T + i_{SS}$. The partial solution i_{SS} of inhomogeneous differential equation $Ldi_{SS}/dt + Ri_{SS} = 0$ is equal to the current value when transient process is over $i_{SS} = 0$

i_T is the general solution of homogeneous differential equation $Ldi_T/dt + Ri_T = 0$. The characteristic equation corresponding to this differential one is $pL + R = 0$ with its root $p = -R/L$. The time constant is $\tau = 1/p = L/R$. Since the characteristic equation has one real root, the transient component is $i_T = A\, e^{pt}$. Constant of integration can be found from initial conditions: $i(0) = i_T(0) + i_{SS}(0) = A$. According to the first commutation law $i(0) = i(0–) = V/R$, so $A = V/R$, $i_T = V/R\, e^{-(R/L)t}$.

The solution of differential equation is (fig.5.6):

$$i = i_T = V/R\, e^{-(R/L)t}.$$

The voltage on resistive element is (fig.5.5):

$$v_R = Ri = R(V/R)e^{-(R/L)t} = Ve^{-(R/L)t}.$$

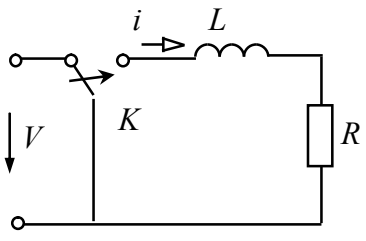

Fig.5.4

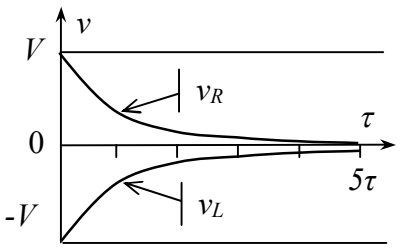

Fig.5.5

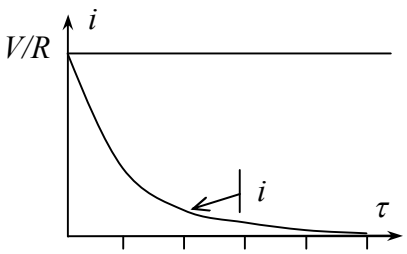

Fig.5.6

The voltage on inductive element is (fig.5.5):

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}\left(\frac{V}{R}e^{-(R/L)t}\right) = L\frac{V}{R}(-\frac{R}{L})\cdot(e^{-(R/L)t}) = -Ve^{-(R/L)t}.$$

Let's analyse the transient process when *RC* link is connected to DC source (fig.5.7). The differential equation according to the Kirchhoff's second law for after commutation steady-state mode is $Ri+v_C=V$, $i=C(dv_C/dt)$, then $RC(dv_C/dt)+v_C=V$. Its solution is $v_C=v_{CT}+u_{CSS}$. The partial solution v_{CSS} of inhomogeneous differential equation $RC(dv_C/dt)+v_C=V$ equals to the voltage value on *C* when the transient process is over. The circuit current equals zero in this case, because the input voltage is applied directly to capacitance $v_{CSS}=V$.

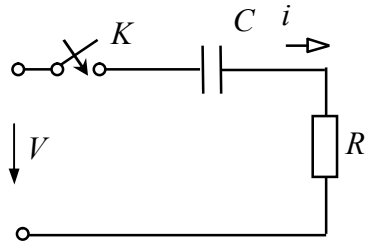

Fig.5.7

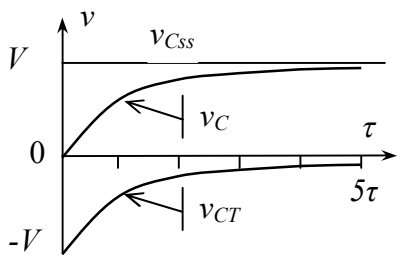

Fig.5.8

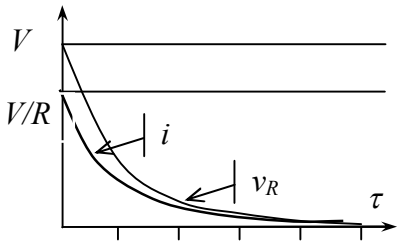

Fig.5.9

v_{CT} is the general solution of homogeneous differential equation $RC(dv_C/dt)+v_C=0$. The characteristic equation corresponding to this differential one is $RCp+1=0$ with its root $p=-1/(RC)$. The time constant is $\tau=1/p=RC$. Since the characteristic equation has one real root, the transient component is $v_{CT}=Ae^{pt}$. Constant of integration can be found from initial conditions: $v_C(0)=v_{CT}(0)+v_{CSS}(0)=A+V$. According to the first commutation law $v_C(0)=v_C(0-)=0$, so $A=-V$, $v_{CT}=-Ve^{-t/RC}$.

The solution of differential equation is (fig.5.8):

$$v_C(t)=V-Ve^{-t/RC}=V(1-e^{-t/RC})=V(1-e^{-t/\tau}).$$

The current is (fig.5.9):

$$i=C(dv_C/dt)=C\frac{d}{dt}\left(V-Ve^{-t/RC}\right)=-CV/(RC)(-e^{-t/RC})=(V/R)e^{-t/RC}.$$

The voltage on resistive element is (fig.5.9):

$$v_R=Ri=R(V/R)(e^{-t/RC})=Ve^{-t/RC}.$$

Let's analyse the transient process when *RC* link is disconnected from DC source and shortened (fig.5.10). The differential equation according to the Kirchhoff's second law for after commutation steady-state mode is: $RC(dv_C/dt)+v_C=0$. Its solution is $v_C=v_{CT}+v_{CSS}$. The partial solution v_{CSS}equals zero, because this equation is homogeneous.

v_{CT} is the general solution of homogeneous differential equation $RC(dv_C/dt)+v_C=0$. The characteristic equation corresponding to this differential one is $RCp+1=0$ with its root $p=-1/(RC)$. The time constant is $\tau=1/p=RC$. Since the characteristic equation has one real root, the transient component is $v_{CT}=Ae^{pt}$. Constant of integration can be found from initial conditions:

$v_C(0) = v_{CT}(0) + v_{CSS}(0) = A$. According to the first commutation law $v_C(0) = v_C(0–) = V$ (the circuit current is equal to zero in this case, because the input voltage is applied directly to the capacitance), $v_{CT} = Ve^{-t/RC}$.

The solution of differential equation is (fig.5.11):

$$v(t) = v_{CT}(t) = Ve^{-t/RC}.$$

The current is (fig.5.12):

$$i = C(dv_C / dt) = C\frac{d}{dt}\left(Ve^{-t/RC}\right) = CV/(RC)(-e^{-t/RC}) = -(V/R)e^{-t/RC}.$$

The voltage on resistive element is (fig.5.12):

$$v_R = Ri = R(-V/R)(e^{-t/RC}) = -Ve^{-t/RC}.$$

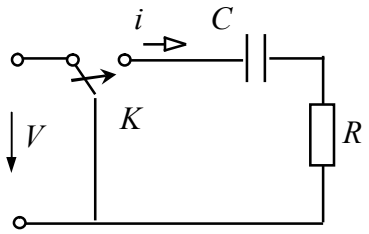

Fig.5.10

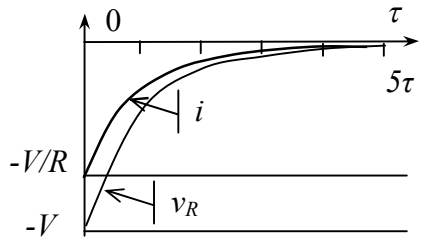

Fig.5.11

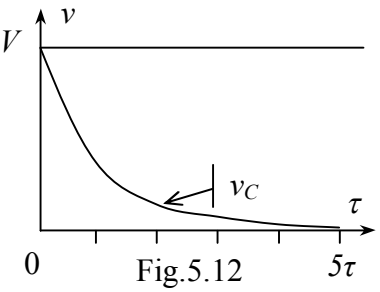

Fig.5.12

Attachment

№	Greek letters		
1	*A*	*α*	*alfa*
2	*B*	*β*	*beta*
3	*Γ*	*γ*	*gamma*
4	*Δ*	*δ*	*delta*
5	*E*	*ε*	*epsilon*
6	*Z*	*ζ*	*dzeta*
7	*H*	*η*	*eta*
8	*Θ*	*θ,*	*teta*
9	*I*	*ι*	*jota*
10	*K*	*κ*	*kapa*
11	*Λ*	*λ*	*lambda*
12	*M*	*μ*	*miu*
13	*N*	*ν*	*niu*
14	*Ξ*	*ξ*	*ksi*
15	*O*	*o*	*micron*
16	*Π*	*π*	*pi*
17	*P*	*ρ*	*ro*
18	*Σ*	*σ,ς*	*sigma*
19	*T*	*τ*	*tau*

20	*Y*	*υ*	*ipsilon*
21	*Φ*	*φ*	*fi*
22	*X*	*χ*	*hi*
23	*Ψ*	*ψ*	*psi*
24	*Ω*	*ω*	*omega*

Physical values designation and units

Value	***Designation***	***Dimension***
Resistance	*R*, Ω	***Om***
Reactance	*X*, Ω	***Om***
Impedance	*Z*, Ω	***Om***
Conductance	*G*, *Sm*	***Simens***
Susceptance	*B*, *Sm*	***Simens***
Admittance	*Y*, *Sm*	***Simens***
Capacity	*C*, *F*	***Farada***
Inductance	*L*, *H*	***Henry***
Inductance mutual	*M*, *H*	***Henry***
Electromotive force	*E*, *V*	***Volt***
Potential	φ, *V*	***Volt***
Voltage	*V*, *V*	***Volt***
Current	*I*, *A*	***Amper***
Active power	*P*, *W*	***Watt***
Reactive power	*Q*, *VAr*	***Volt-Amper reactive***
Total power	*S*, *VA*	***Volt-Amper***
Magnetomotive force	*F*, *A*	***Amper***
Magnetic induction	*B*, *T*	***Tesla***
Magnetic field tension	*H*, *A/m*	***Amper per meter***
Magnetic stream	Φ, *Wb*	***Weber***
Linkage	ψ, *Wb*	***Weber***
Magnetic permeability (absolute)	μ_a,, *Гн/м*	***Henry per meter***
Magnetic permeability (relative)	μ	
Magnetic constant	μ_0, *Гн/м*	$4\pi \cdot 10^{7}$
Frequency	*f*, *Hz*	***Herz***
Angular frequency	ω, *rad/s*	***radian per second***
Length	*l*, *m*	***meter***
Hight, depth	*h*, *m*	***meter***
Layer	δ, *d*, *m*	***meter***
Arial	*S*, m^2	***square meter***
Magnetic resistance	R_m	
Number of turns	*w*	
Force	*F*, *N*	***Newton***
Work (energy)	*W*, *J*	***Joule***
Charge	*Q*, *C*	***Coulomb***

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