

**NUMERICAL SIMULATION OF CRACK TIP OPENING AT STATIC AND  
DYNAMIC CREEP**

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**ABSTRACT**

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The paper proposes finite element model for studying of influence of brittle inclusions in the material on opening in the crack tip in the conditions of static and dynamic creep, when the parameters of structural heterogeneousness is taken into consideration. Using this model, there have been studied the influence of shape ratio and specific part of inclusions of heterogeneous material on the crack tip opening increase and material damage at static and dynamic creep.

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During monotonously increasing loading of elasto-plastic body with a sharp crack, there occurs opening of its tip to the particular point  $\delta_k$ , on reaching which the crack begins to spread. In conditions of creep of cracked body the gradual increase of its opening takes place as a results of creep processes. In the proximity of crack front, because of large stress in the material, the cavities, gaps, submicrocracks, etc. are formed which is denoted by the trajectory of its growth during further expansion.

The crack tip opening velocity during creep depends much on loading applied. In particular, the cyclic component of loading noticeably influences on both deformation of the materials and microstructural changes [1]. During dynamic creep repeated change of loading from minimum to maximum point takes place, and is followed by the increase of plastic deformation [2]. Plastic deformation storage at creep conditions depends on loading regime while cyclic component noticeably decreases durability in comparison with static loading [2,3].

At heterogeneous alloy creep in its components (inclusion or matrix) the ultimate state can be reached, and as the reason of this, in the local parts of the components are fractured. This causes strain redistribution and deformations in the material. Finite element method (FEM) simulation of inclusion influence on the crack growth trajectory in composite with aluminum matrix was made in works [4,5].

Modeling of inclusions clustering influence aluminum alloy damage, void and microcracks formation is calculated in [6]. It was shown that the failure stress of composites increases with increasing the average nearest-neighbor distance between the particles in the composite, and with decreasing the degree of clustering of particles.

The modeling of size and mechanical property features of inclusion on crack growth is made in [7-9]. Numerical results for an edge-cracked, graded specimen show that the particle shape and orientation for the same phase volume fractions have negligible effects on fracture reliability, even for graded materials with a high modular ratio [8].

The influence of the specific part and inclusions shape ratio on crack tip opening at the creep and dynamic creep conditions is not studied well.

In this article the FEM modeling of brittle inclusions fracture in the heterogeneous material on crack tip opening in the conditions of creep and dynamic creep is made, when the parameters of specific part and inclusions shape ratio is taken into consideration.

**Methods of investigation**

For studying of influence of brittle inclusions fracture in the material on opening in the crack tip at the conditions of static creep and dynamic creep, the FEM model was created (Fig.1). In the crack tip structurally not homogenous elements are situated. The model consists of three components: plastic matrix, brittle inclusion, that are placed in matrix according to two-dimension normal law of

distribution and material itself, which is modeled (Fig.1). The inclusions are oriented in the direction of loading application.

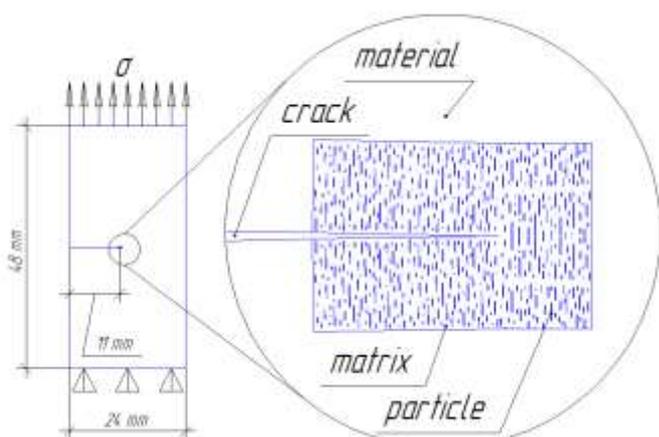


Fig.1. Calculation model with crack with structurally heterogeneous block in the crack tip.

of the model, the lower line was fixed, and its vertical movement was limited (Fig.1). Finite element net for the models was created by means of two-dimensional element plane 82 [11]. The element has the qualities of quadratic displacements representation and is used for modeling with irregular net of finite elements. It is denoted by eight nodes, that have two states of freedom in each node. The element has the features of plasticity, hyperelasticity, creep, hardness, increase at existence of loading, noticeable displacements and strains. The element can take quadrangular and triangular shape. The modelling was made in the plane strain conditions. For calculations the option of matrix and inclusions fracture was activated in the model. Fracture was made by method, described in work [12], when unsteadiness of tension fields and deformations in the crack tip was taken into account.

Creep was modelled at constant stress intensity factor (SIF)  $K_s$  (curve 1, Fig. 3). Dynamic creep was modelled with implementation of high-frequency ( $f = 25$  Hz) and low-amplitude ( $K_a = \pm 1,1 \text{ MPa} \sqrt{m}$ ) component (curve 2) on constant loading. The meaning of maximum SIF at dynamic creep conditions  $K_{\max}$  was ensured, when  $K_s = K_{\max} = 31,1 \text{ MPa} \sqrt{m}$ .

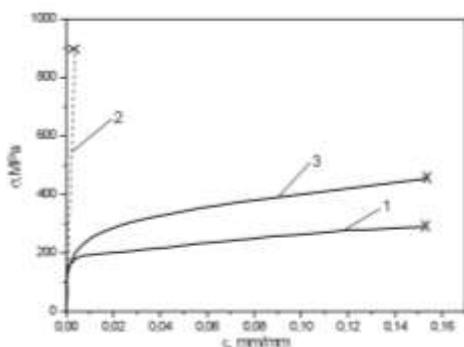


Fig.2. Diagrams of deformation for matrix (1), inclusions (2) and total diagram for strain of the material (3).

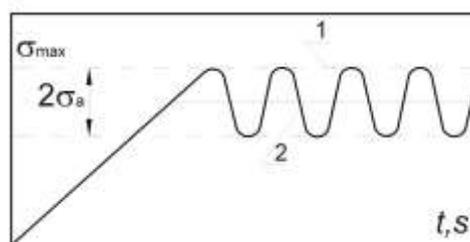


Fig. 3. Scheme of loading during testing: 1 – static creep; 2 – dynamic creep.

For studying of influence of structural unhomogeneous parameters (specific part (S) and shape ratio ( $\alpha$ ) of inclusions) on crack tip opening and strength at creep and dynamic creep conditions, in software complex ANSYS two groups of finite element models were create.

The first group of models (Fig.4) was used for studying of influence specific part of inclusions S on crack tip opening. In all four models of this group the inclusions size was not changed: inclusion diameter  $d = 0,1 \mu\text{m}$ , inclusion length  $l = 0,8 \mu\text{m}$ .

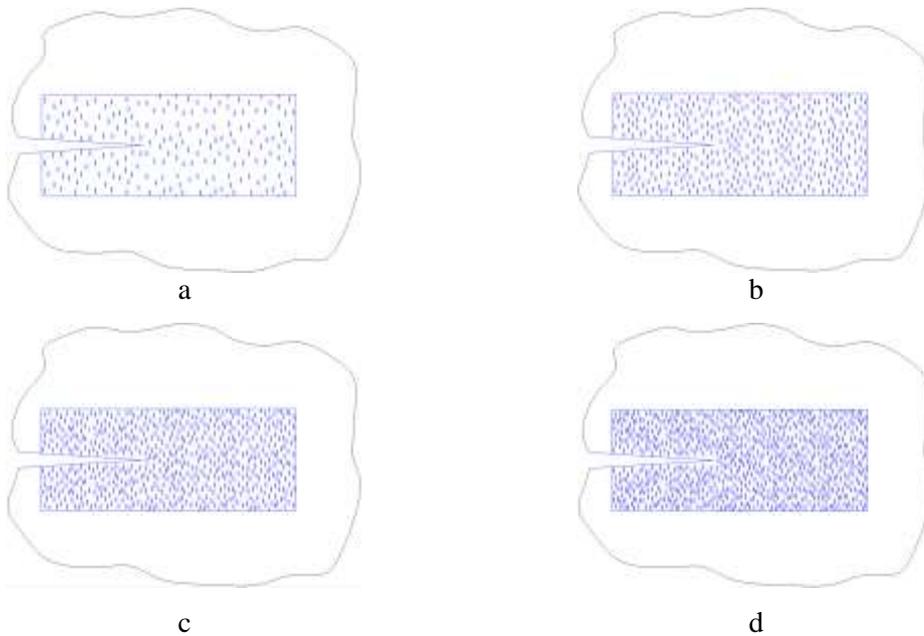


Fig. 4. Calculation models for studying of specific part influence on crack tip opening:  
a –  $S=3\%$ ; b –  $S=6\%$ ; c –  $S=9\%$ ; d –  $S=12\%$ .

The influence of inclusion shape ratio on crack tip opening was studied on the second model group. In all models the specific part ( $S=6\%$ ) and the diameter of inclusion ( $d = 0,1\mu\text{m}$ ) remained unchanged.

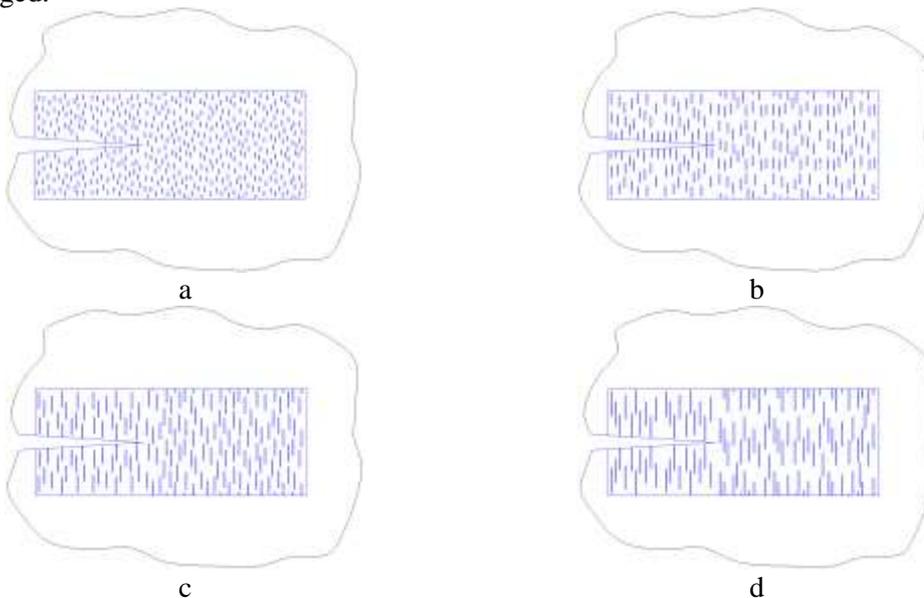


Fig. 5. Calculation models for studying of influence of inclusion shape ratio on crack tip opening: a –  $\alpha = 8$ ; b –  $\alpha = 16$ ; c –  $\alpha = 25$ ; d –  $\alpha = 36$ .

The model loading has grown iteratively ranging from 0 to  $31,1 \text{ MPa} \sqrt{m}$  with the iteration step  $0,1 \text{ MPa} \sqrt{m}$ . On every loading step, the condition of inclusions and matrix fracture was checked and elements, which satisfied that conditions (critical strains of matrix fracture  $\sigma_f^{matrix} = 825 \text{ MPa}$ , critical strains of inclusions fracture  $\sigma_f^{particles} = 1100 \text{ MPa}$  [12]) were deactivated.

For crack tip opening calculation, specially created post-processor macroses were used on every iterational step of loading. Application of these macroses gives the possibility to automate receiving the calculation results and their working out. During calculations damage of the simulation material in around the crack tip in the models of the first and second group at static and dynamic loading, has been studied. For this purpose in the area of critical strain of inclusion fracture (Fig. 6), the area of all voids, which appeared as a result of structural component fracture, was found.

The relation of these voids size to the area analyzed denotes damage of the material ( $\omega$ ). The size of structurally unhomogenous block in the crack tip was defined by the area on which normal strains are equal to those of inclusions fracture.

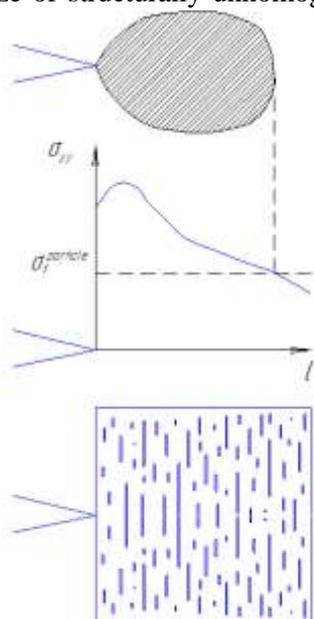


Fig. 6. Area of material damage measurement.

To calculate the crack tip opening the known EMP method [13] with taking into consideration the real strain, was used. It is based on supposition that material deformation in the crack creep tip can be modeled by the smooth specimen creep with a length  $L_{ref}$  at uniaxial stretch with strain  $\sigma_{ref}$ :

$$\sigma_{ref} = \frac{P \cdot \sigma_{0,2}}{P_{0,2}}, \quad (1)$$

where  $\sigma_{0,2}$  - yield stress;  $P_{0,2}$  - strength at deformation 0,2%.

The length of the conventional smooth specimen was taken as proportional to the width of remaining undestroyed part of a cracked specimen:

$$L_{ref} = \gamma (b - l), \quad (2)$$

where  $\gamma$  - ratio;  $b$  - specimen width;  $l$  - crack length.

The  $\gamma$  ratio was defined from the terms that length increase  $\Delta L_{ref}$  of conventional specimen was equal to smooth specimen crack tip opening increase because of creep:

$$\Delta L_{ref} = \Delta \delta. \quad (3)$$

Creep strain increase of the smooth specimen  $\Delta L_{ref}/L_{ref} = p$  is satisfactorily described by relation of creep strain on time, when shape ratio change and specific part of inclusions are taken into consideration [14]:

$$p = \left( \frac{\sigma_{ref}}{1 - \omega} \right)^{C_1} t^{C_2} \alpha^{C_3} (C_4 + C_5 S), \quad (4)$$

where  $C_1, C_2, C_3, C_4, C_5$  - constant.

On the basis of equation (4) increase of opening  $\Delta \delta$  in the creep conditions is written in this way:

$$\Delta \delta = \left( \frac{\sigma_{ref}}{1 - \omega} \right)^{C_1} t^{C_2} \alpha^{C_3} (C_4 + C_5 S) \cdot L_{ref}. \quad (5)$$

### Results of calculations and their discussing

On Fig. 7 the fragments of models in the creep (a) and dynamic creep (b) conditions at equal material damage (total area of voids that appeared) are shown. The quantity and voids size considerably depends on type of loading applied to the calculation model. At creep, big voids are formed but their number is not large (Fig.7a). Vice versa, at dynamic creep small sized voids appear, but their quantity is larger (Fig.7b).

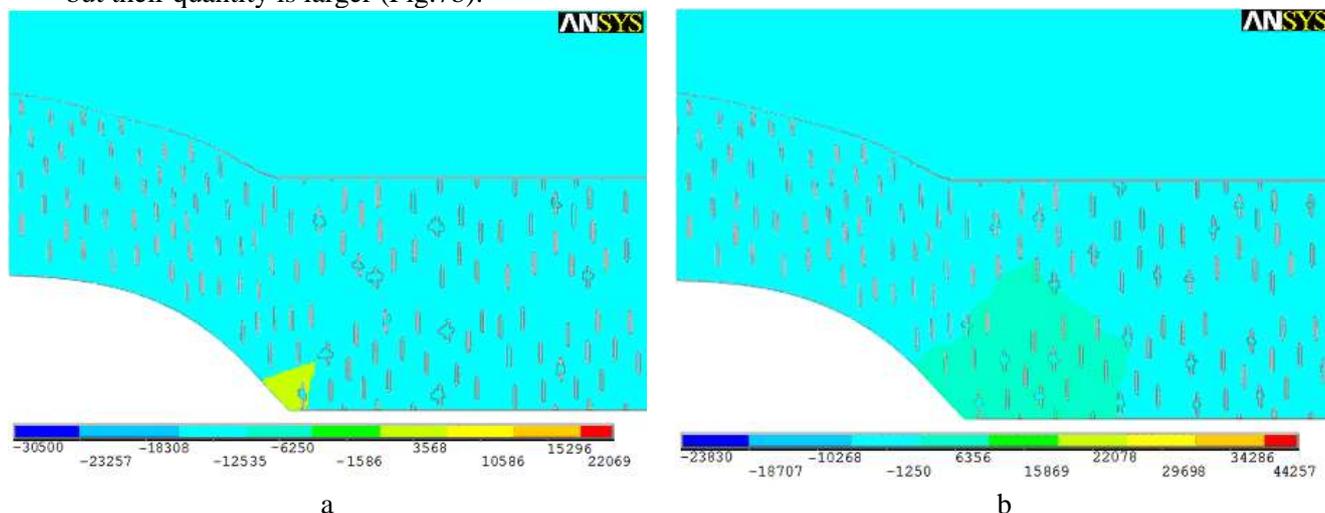


Fig. 7. Material damage in the crack tip at: a - creep; b - dynamic creep.

During simulation of creep and dynamic creep processes, damage of material model was considered and on every iterational step in the time of loading opening in the crack tip has been measured. On Fig. 8 there are the dependence of crack tip opening to loading time in the conditions of creep and dynamic creep at different specific part of inclusions and shape ratio. Cyclic component of the loading causes bigger crack tip opening in comparison with static loading at constant SIF  $K_{max}$ . When the specific part of inclusions increases (Fig.8a) strength of the material grows and plasticity in around the crack is getting less. It causes crack tip opening decrease at equal loading time. Inclusions shape ratio growth at constant diameter causes the inclusions length growth and material armation, as a results, the crack tip opening is getting less at the same time of loading application. Formula (5) was used to describe the crack tip opening increase in time  $\Delta\delta$  at creep and dynamic creep. When  $\gamma=0,18$  [15], stated for every model (S,  $\alpha$  - const),  $L_{ref}$  and  $\sigma_{ref}$  are found. Damage ( $\omega$ ) was defined in the equal time (5 min) according to the methods described. On Fig.8 the results, received by FEM and recalculation by formula (5) with material damage taken into consideration, are shown. Accuracy between the calculation data by formula (5) and results received by FEM is not bigger than 12,5%.

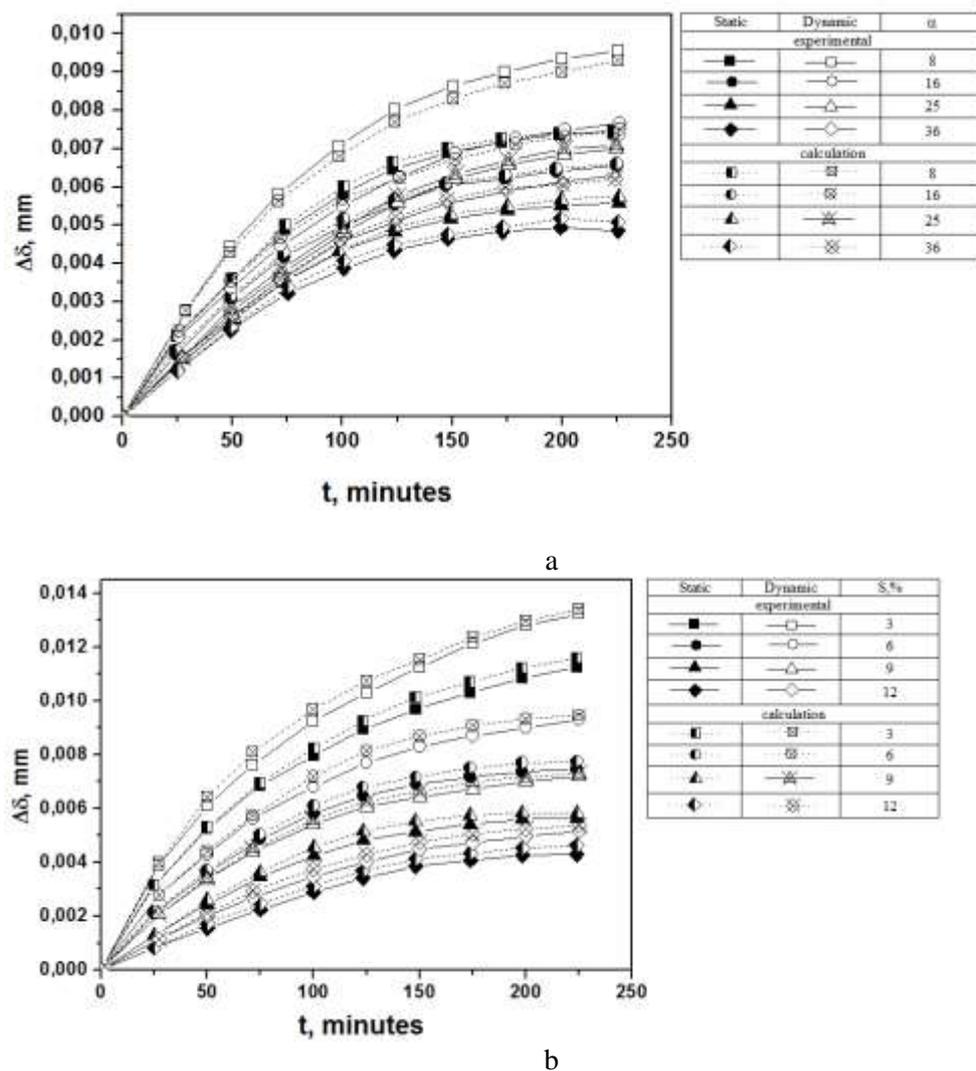


Fig. 8. Relation crack tip opening  $\Delta\delta$  to loading time  $t$  at creep and dynamic creep: a – at different inclusions shape ratio; b – at different specific part of inclusions.

### Conclusions

The finite element model for studying of influence of brittle inclusions in the material on opening in the crack tip in the conditions of static and dynamic creep is made, when the parameters of structural heterogeneity is taken into consideration. Using this model the influence of shape ratio and specific part of inclusions of heterogeneous material on the crack tip opening increase and material damage at static and dynamic creep was studied.

It is stated that within the increase of the specific part and inclusions shape ratio the increase of crack tip opening in the conditions of static and dynamic creep is decreased. It should be noted that dynamic creep is followed by larger crack tip opening than at static one.

It was found that at the same damage of heterogeneous material in the crack tip at static creep small amount of voids with larger geometric parameters is formed, and at dynamic creep bigger amount of small sized voids is observed.

The methods of crack tip opening increase calculation on the basis of EMP method is proposed with taking into consideration the damage of the material in the conditions of static and dynamic creep which includes shape ratio and specific part of inclusions of heterogeneous material. The results, received by EMP method and the proposed finite element model, have been compared.

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