Experiment 8

DETERMINATION OF LOGARITHMIC DECREMENT AND DAMPING COEFFICIENT OF OSCILLATIONS

Purpose of of the experiment: to master the basic concepts of theory of oscillations. To determine logarithmic decrement and damping coefficient.

1 EQUIPMENT

1. Pendulum with a scale.
2. Stop-watch.

2 THEORY

2.1 Harmonic oscillations are the variations of physical quantities in time, governed by low of sine or cosine:

$$a = A\cos(w_0 t + j_0), \tag{2.1}$$

where α is a value of the varying quantity at the moment of time t, A is an amplitude of oscillations (the maximal value of physical quantity), $(w_0 t + j_0)$ is a phase of oscillations (it determines deviation from equilibrium position), j_0 is an initial phase (it determines deviation at the moment of time t=0), w_0 is angular frequency.

Period of harmonic oscillations T is the time, required for completion of one full oscillation:

$$T = \frac{2p}{w_0} = \frac{1}{n} \qquad [T]=1c \tag{2.2}$$

ν is frequency of oscillations (number of oscillations per second). Unit of frequency is 1 Hz (cycle per second).

Harmonic oscillations can take place only under action of resilient forces (or other force which return the system to the equilibrium state and is proportional to the deviation from equilibrium $F = -ka$, where α is deviation from equilibrium, k is coefficient of proportionality).

2.2 In this experiment the oscillations are studied on example of physical pendulum (Fig. 2.1). Physical pendulum is a body, that oscillates about a horizontal axis under action of force, that does not passes through the center of the masses of the body.

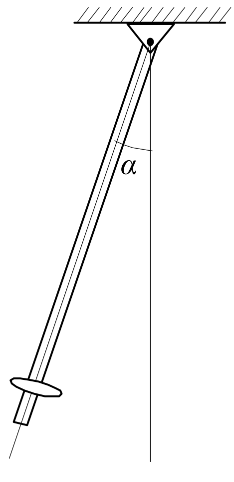

Рисунок 2.1

We may formulate the law of motion of physical pendulum on the basis energy conservation law.

In the arbitrary moment of time sum of kinetic and potential energies of pendulum equals

$$E = \frac{Iw^2}{2} + \frac{ka^2}{2}, \tag{2.3}$$

where I is moment of inertia of pendulum about an axis, that passes through the point of suspension w is angular velocity of pendulum at the given instant, m is mass of pendulum, g is acceleration of the free falling, d is distance from the axis of rotation of pendulum to the center of mass, a is an angle of deviation of pendulum from equilibrium position. Reduction of energy of the system dE during the displacement of pendulum on the corner da, caused by the loss of energy for overcoming the forces of friction

$$dE = dA,$$

where elementary work of moment of friction forces M_F is

$$dA = -M_F da, \tag{2.4}$$

the “minus” sign means that the force of friction leads to the decrease of the system energies ($dE<0$).

Suppose that moment of force of friction is proportional to angular speed:

$$M_F = rw, \tag{2.5}$$

where r is coefficient characterizing friction, w is angular velocity ($w = \frac{da}{dt}$). Relation (2.5) comes from the analogy with forces of friction at the motion of material point (for example, mathematical pendulum).

As we have

$$dE = Iwdw + kada, \tag{2.6}$$

then

$$Iwdw + kada = -rwda.$$

Forasmuch

$$I\frac{dw}{dt} + ka + r\frac{da}{dt} = 0, \tag{2.7}$$

or, taking into account, that

$$\frac{dw}{dt} = \frac{d^2a}{dt^2}, \quad \frac{da}{dt} = w,$$

we have

$$\frac{d^2a}{dt^2} + 2b\frac{da}{dt} + w_0^2 a = 0, \tag{2.8}$$

where notations $b = \frac{r}{2I}$ and $w_0^2 = \frac{k}{I}$ have been introduced. Quantity b is called the damping coefficient of oscillations, w_0 is angular eigenfrequency of oscillations.

The equation (2.8) has solution in the form:

$$a = A(t)\cos(wt + j_0), \tag{2.9}$$

which describes damped oscillations. Here $A(t)$ is amplitude of damped oscillations:

$$A(t) = A_0 \exp(-bt), \tag{2.10}$$

A_0 is initial amplitude (at the moment $t=0$), w is angular frequency of damped oscillations

$$w^2 = w_0^2 - b^2. \tag{2.11}$$

The period of damped oscillations is given by formula

$$\mathrm{T} = \frac{2p}{w} = \frac{2p}{\sqrt{w_0^2 - b^2}}. \tag{2.12}$$

Relations (2.9)-(2.12) take place only at condition $w_0^2 - b^2 > 0$, if $b^2 < w_0^2$ then oscillations do not appear because of significant resistance of environment.

3 DERIVATION OF COMPUTATION FORMULA

Let the amplitude of damped oscillations at the moment t_1:

$$A_1 = A_0 e^{-bt_1}, \qquad (3.1)$$

and corresponding amplitude at the moment t_2:

$$A_2 = A_0 e^{-bt_2}. \qquad (3.2)$$

Divide (3.1) on (3.2) and get:

$$\frac{A_1}{A_2} = e^{-b(t_1-t_2)} = e^{b(t_2-t_1)} = e^{b\Delta t} = e^{bn\mathrm{T}}, \qquad (3.3)$$

where n is number of complete oscillations made in time $\Delta t = t_2 - t_1$, $T = \frac{\Delta t}{n}$ is period of these oscillations. Taking logarithm of equation (3.3), we get

$$\ln\frac{A_1}{A_2} = bnT.$$

Where from

$$b = \frac{1}{nT}\ln\frac{A_1}{A_2} = \frac{1}{t}\ln\frac{A_1}{A_2}. \qquad (3.4)$$

This equation means that damping coefficient is a quantity, reciprocal to the time, during which amplitude is reduced by factor e (ln(A_1/A_2)=ln(e)=1).

The logarithmic decrement of the oscillation damping is the quantity

$$l = bT = \frac{1}{n}\ln\frac{A_1}{A_2}. \qquad (3.5)$$

From a formula (3.5) it follows that logarithmic decrement is a quantity, reciprocal to the number of oscillations, during which amplitude is reduced by the factor e.

4 APPARATUS

The apparatus (Fig. 4.1) consists of physical pendulum, that can oscillate about the fixed axis and the electronic block for measuring the number of oscillations and their total time. The period of oscillation for the pendulum used can be changed by changing the position of the load on the bar. To start experiment one should to deviate the pendulum from position of equilibrium (to measure a deviation, angular degrees are marked on the scale), then clear the readings of the electronic block (pushing the button "CLEAR") and to release the pendulum. A photogate fixes the number of oscillations N and total time of oscillations t. Using these data it is possible to find the period of oscillations T=t/N. To stop counting oscillations at some moment (for example, at tenths oscillation), one should push the button of "STOP" during the last oscillation (in the above example, when reading of the electronic block is nine).

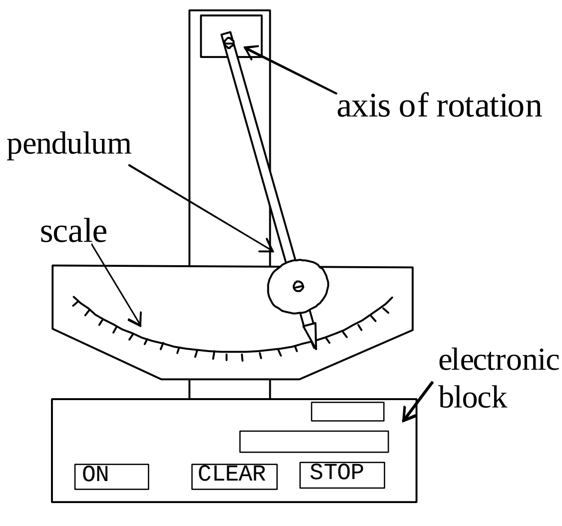

Figure 4.1