Experiment 4

DETERMINATION OF LIQUID VISCOSITY BY STOCKES METHOD

Objective of the experiment: To master the basic concepts of fluid mechanics and to determine the dynamic coefficient of viscosity for a liquid.

1 EQUIPMENT

1. Stockes' apparatus.
2. Metallic ball.
3. Micrometer.
4. Vernier caliper.
5. Stop-watch.

2 THEORY

2.1 In many practically important cases, the fluid mechanics based on the notion of an ideal liquid is not applicable. In distinction from the ideal liquid, in real ones there exist the forces tangential to the surface of moving layers contact. These forces are called viscous friction (internal friction) forces or viscous forces.

Viscosity is the property of a fluid (liquid or dense gas) to offer resistance at the relative motion of their layers.

2.2 In the stream of the real liquid in the close vicinity of solids, moistened by it, the layers move with different velocities. The layer which touches a solid directly has zero velocity. The further a layer is from the resting solid, the larger is the value of its velocity.

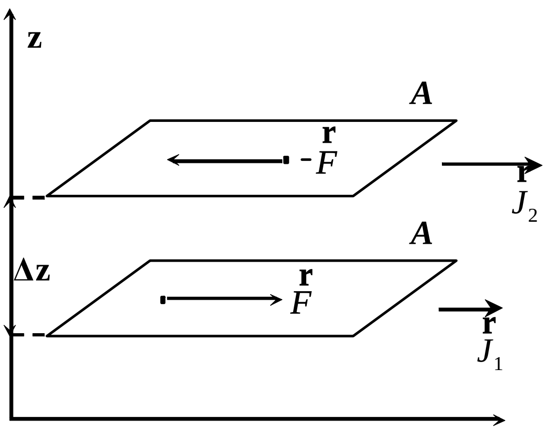

Figure 2.1

Let us imagine the stream of liquid to be composed of infinitely many layers. Friction forces appear when the layers start moving relatively to each other. Let us choose two parallel layers in a stream of liquid with equal areas A (see Fig. 2.1). Let Δz denote the distance between the layers. Experiments show that tangent friction forces act on every layer, the force acting on the layer, which moves slower, is directed along the velocity and that acting on the faster layer is opposite to the velocity. The magnitude of this viscous force is given by the law of Newton

$$F = hA\left|\frac{\Delta J}{\Delta z}\right|, \qquad (2.1)$$

where h denotes the coefficient of dynamic viscosity (or coefficient of internal friction); A is the area of layer surface; $\left|\frac{\Delta J}{\Delta z}\right|$ is the rate of velocity change in the direction, perpendicular to the flow direction.

2.3 Using equation (2.1), we have

$$h = \frac{F}{\left|\frac{\Delta J}{\Delta z}\right| A} \cdot \qquad (2.2)$$

The coefficient of dynamic viscosity is quantitatively equal to force of friction, that acts on unit of layer surface, if change of speed in perpendicular direction is equal to 1m/s on 1 m of distance. In the SI system of units the viscosity is measured in Pa·s

$$[h] = \frac{1\,\mathrm{N}}{\frac{1\frac{\mathrm{m}}{\mathrm{s}}}{1\,\mathrm{m}} \cdot 1\,\mathrm{m}^2} = 1\frac{\mathrm{N}\cdot\mathrm{s}}{\mathrm{m}^2} = 1\mathrm{Pa}\cdot\mathrm{s}.$$

Viscosity of liquids strongly depends on temperature. At the rise of temperature the viscosity coefficient decreases.

2.4 Let us consider motion of symmetric body in the real liquid. As a result of viscosity the liquid can not slide freely along the surface of body, very thin layer of liquid covers the surface of body and moves with it as a whole. Therefore, there are forces of friction only between the layers of liquid, not between the solid and the liquid, so these forces does not depend on material of the body and are determined only by body's shape and properties of liquid.

Experiments show that the value of resulting force of viscosity, exerted on a body, is proportional to the value of body's velocity (if value of velocity is small enough)

$$F = CJ\,. \qquad (2.3)$$

The coefficient of proportionality C depends on the shape of body, its characteristic dimensions, orientation in the stream of liquid and properties of the liquid.

In front of the moving body the pressure of fluid is larger than one behind the body, so the force acts on the body in opposite direction to its velocity. The value of this drag force equals

$$F_{л.оп.} = C_1 J^2\,. \qquad (2.4)$$

Here C_1 is a coefficient of proportionality,which also depends on the body's shape, its characteristic dimensions, orientation in the stream of liquid and properties of the liquid.

Consequently, both viscous friction force and drag force act simultaneously on a symmetric body in the stream. Forces of viscous friction are applied to lateral surface of body, drag forces are applied to the fore-part of the body, it depends on speed of the specific stream, which forces have larger influence. In this experiment relative velocities are small, so grad forces may be neglected.

3 DERIVATION OF COMPUTATION FORMULA

One of methods for determination of dynamic viscosity coefficient h is Stockes method, based on measuring of speed of uniform motion of small spherical body (ball) in the liquid under investigation.

In the case of small speed (when a flow is laminar one) by low of Stockes the force of viscous friction F (see formula 2.3) is proportional to the coefficient of viscosity, radius of the ball r and velocity of the ball J

$$F = 6phrJ\,. \qquad (3.1)$$

Let us consider the falling of ball in a stationary liquid. Three forces are applied to the falling ball (Figure 3.1), namely the gravity force, upward buoyant (Archimedes') force $\vec{F}_A$ and the force of viscous (internal) friction $\vec{F}$, directed against motion of the ball. At first the ball moves with constant acceleration, however with the increase of speed force of viscous friction increases and reaches a value large enough to balance all other acting forces (see Figure 3.1)

$$mg = F + F_A . \qquad (3.2)$$

The buoyant force is given by expression:

$$F_A = m_L g = V r_L g = \frac{4}{3} p r^3 r_L g , \qquad (3.3)$$

where m_L is mass of liquid in the volume of the ball; r_L is density of the liquid; *V* is volume of marble; g stands for the free fall acceleration.

Gravity force for the ball can be written as

$$m_b g = V r_b g = \frac{4}{3} p r^3 r_b g , \qquad (3.4)$$

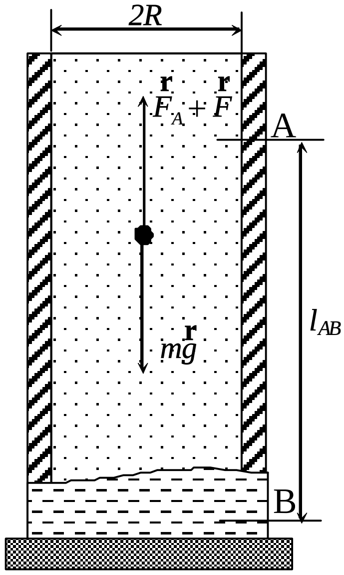

Figure 3.1

where r_b denotes density of the body. Taking into account expressions for forces (3.1), (3.3) and (3.4) in formula (3.2), we get

$$\frac{4}{3} p r^3 r_b g = 6prhJ + \frac{4}{3} p r^3 r_L g , \qquad (3.5)$$

from where we obtain for the coefficient of dynamic viscosity

$$h = \frac{2}{9} \frac{r^2 (r_b - r_L) g}{J} . \qquad (3.6)$$

Stockes' law (3.1) and formula (3.6) are valid in the case, when a body moves uniformly, without the rotation or turbulence, in a homogeneous liquid which has no bounds.

In this experiment the ball falls in a long cylinder with an internal radius ***R***. Taking into account finite dimensions of the cylinder we have to rewrite formula (3.6) as

$$h = \frac{2}{9} \frac{r^2 (r_b - r_L) g}{J (1 + 2,4 \frac{r}{R})} . \qquad (3.7)$$

Apparatus to be used for determination of viscosity coefficient is composed by of the long cylinder filled with a liquid. There are two marks on the cylinder, A and B (Figure 3.1). An overhead mark is made on such a distance from the upper level of liquid, that a ball having passed this distance moves uniformly. The distance l_{AB} between marks A that B is passed with the constant velocity

$$J = \frac{l_{AB}}{t} , \qquad (3.8)$$

where ***t*** is the time of body's sinking from one mark to the other.

Substituting (3.6) in (3.5) and taking into account, that in experiment not radii but the diameter of the ball ***d*** and internal diameter ***D*** of the cylinder will be measured, we get a computation formula

$$h = \frac{1}{18} \frac{d^2 (r_b - r_L) g t}{l_{AB} (1 + 2,4 \frac{d}{D})} . \qquad (3.9)$$