Experiment 3

STUDY OF ROTATIONAL MOTION OF RIGID BODY ON OBERBEK PENDULUM

Purpose of the experiment: to master the basic concepts of kinematics and dynamics of rigid body rotational motion. To determine the moment of inertia for Oberbek pendulum.

1 EQUIPMENT

1. Oberbek pendulum.
2. Millimeter scale.
3. Stop-watch.
4. Vernier caliper.

2 THEORY

2.1 In rotational motion of rigid body about the fixed axis all points of the body move on circular trajectories, the centers of which lie on the same line called the rotation axis. Circles, which the points of body move on, lie in planes perpendicular to the axis of rotation. Points of a body, which lie on the axis of rotation are immobile. Fig 2.1 illustrates rotational motion of a cross-piece with weights mounted on it, under the action of tension force of a string. One end of the string is fastened on the pulley of radius *r*, the weight of mass *m* is suspended on the other end. In the figure shown the tension force of the string $\vec{F}$, exerted on the pulley and the force $\vec{T}$, acting on the weight *m* from the side of the string, as well as gravity force $m\vec{g}$.

In rotational motion of rigid body about a fixed axis a position of the body is characterized by angle of rotation *φ*, which is an analog of a path in translational motion. The moving points of a body all have the same angular velocity and angular acceleration.

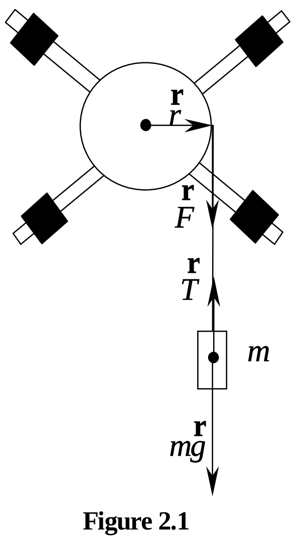


Figure 2.1

We will define the vector of angular velocity $\vec{\omega}$ as

$$\vec{\omega} = \frac{d\varphi}{dt}\vec{k}, \qquad (2.1)$$

where $d\varphi$ is the turn angle made during the time interval *dt*, $\vec{k}$ is a unit vector ($|\vec{k}| = 1$), directed in a positive direction of OZ axis (axis of rotation). By convention, the positive direction is chosen to coincide with the direction of gimlet (drill) motion when its handle is revolved clockwise (this direction is considered positive for angle $d\varphi$). This rule is called the gimlet rule. For the case shown in Fig. 2.1, rods, pulley, weights on the cross-piece, all have the same angular velocity which is directed along the axis of rotation, perpendicular to the plane of the figure. A projection of vector $\vec{\omega}$ on axis OZ equals $\omega = \frac{d\varphi}{dt}$ (magnitude of angular velocity). Unit of angular velocity is 1 rad/s.

Due to the interaction of the body with surrounding bodies the angular velocity of the body can change; this change is characterized by angular acceleration which we will define as

$$\vec{\varepsilon} = \frac{d\omega}{dt}\vec{k} = \frac{d^2\varphi}{dt^2}\vec{k} \qquad (2.2)$$

A vector $\vec{e}$ has direction of vector $\vec{w}$ if magnitude of angular velocity increases ($d|\vec{w}|/dt > 0$) and the opposite direction, if w decreases ($d|\vec{w}|/dt < 0$). For the case shown in Fig. 2.1 the direction of angular acceleration of the system coincides with direction of angular velocity. A projection of $\vec{e}$ on an axis OZ (magnitude of vector $\vec{e}$) equals $e = \frac{dw}{dt} = \frac{d^2 j}{dt^2}$. Unit of angular acceleration is 1 rad/s^2.

Linear kinematic quantities characterizing the specific point of a body (path *s*, velocity J, tangential acceleration a_t) are related to the respective angular characteristics by equations

$$s = j \cdot r, \quad J = w \cdot r, \quad a_t = e \cdot r, \tag{2.3}$$

(*r* – radius of a circle, which the given point of body moves on at a given moment).

2.2 Basic concepts of dynamics of rotational motion of a rigid body are moment of inertia and a torque.

2.2.1 Moment of inertia *I* plays the same role in rotational motion as mass does in translational one. It means that is the moment of inertia is a measure of body's inertia in rotational motion. One can see this from comparison of expression for kinetic energy in rotational motion of a body about a fixed axis ($Iw^2/2$, where *I* denotes moment of inertia and *w* stands for angular velocity) with expression for kinetic energy in translational motion of a body ($mJ^2/2$).

The moment of inertia of a body of arbitrary geometrical form about some axis can be calculated using following equation

$$I = \sum_i \Delta m_i r_i^2, \tag{2.4}$$

stating that the moment of inertia of a rigid body about an axis equals to the sum of products of elementary masses (material points) and the square of their distances to the axis Unit of moment of inertia is 1 kg·m^2.

One can see that the value of moment of inertia depends on mass of the body, its size, form and the choice of rotation axis. Moment of inertia is additive quantity, that is for the system, that consists of a few bodies, a total moment of inertia equals to the sum of moments of inertia of individual bodies.

In particular, the moment of inertia of a particle (material point) about any axis of rotation is:

$$I = mr^2, \tag{2.6}$$

where ***m*** is mass of the particle point, ***r*** is the distance to the axis of rotation. Using the same formula it is possible to calculate the moment of inertia of body on condition that distance from the axis of rotation to the center of mass is much larger of the characteristic linear dimensions of the body.

Moment of inertia of a rod about an axis which passes through the center of the rod bar, athwart to it equals

$$I = \frac{1}{12} ml^2, \tag{2.8}$$

where ***m*** is mass of the rod, ***l*** is its length.

It is possible to determine the moment of inertia experimentally with the help of fundamental law of dynamics of rotational motion of a rigid body.

2.2.2 A body can rotate around a fixed axis on the condition that there is an external force $\vec{F}$ (or a component of a force), in plane perpendicular to the axis, acting on the body. The rotational effect of force $\vec{F}$ is characterized by a physical quantity named torque. A torque in rotational motion plays the same role as force does in translational motion. One may calculate torques about an axis or about a point, these differ in general case.

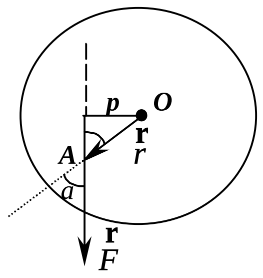


Figure 2.2

We choose the point O on the axis of rotation in the force $\vec{F}$ plane of action (see Fig. 2.2). Then torque of the force $\vec{F}$ with about point O is a vector equal to a cross product of the radius-vector drawn from a point O to the point where the force is applied, and vector of force $\vec{F}$. Module of moment of force:

$$\left|\vec{M}\right| = Fr\sin a = Fp\,, \tag{2.9}$$

where a is angle between vectors $\vec{r}$ and $\vec{F}$, and $p = r\sin a$ is the length of the perpendicular dropped from the axis of rotation on the line of action of the force, this perpendicular is called lever arm.

The magnitude of the torque about fixed axis OZ is the projection of torque $\vec{M}$ about the point O onto an axis OZ:

$$M_Z = Fr\sin a\,. \tag{2.10}$$

Torque about the axis, itself, is defined as a vector quantity

$$\vec{M} = M_Z \cdot \vec{k}\,,$$

where $\vec{k}$ is the unit vector directed along the axis OZ (the indice z may be omitted).

If there are several forces acting on a body, resulting (net) torque about a point O equals the vector sum of component torques. A magnitude of the torque about an axis is the algebraic sum of projections of component torques.

Unit of a torque is N·m.

2.3 A relation between a torque acting on a body, its angular acceleration and moment of inertia is given by the fundamental law of dynamics of rotational motion (also called the second Newton's law for rotational motion):

$$\vec{M} = I\vec{e}\,, \tag{2.11}$$

where $\vec{M}$ is net torque. After projecting onto the axis of rotation we have

$$M = Ie\,,$$

where M is a magnitude of net torque (algebraic sum of torques), which is the torque projection on the axis of rotation, torque about the axis of rotation.

For the case shown in Fig. 2.1, if we neglect friction forces the magnitude of momentum is $M = F \cdot r$ torque is directed along the angular velocity. Having M and e given, using a formula (2.11) it is possible to find out the moment of inertia of the system composed of cross-piece with weights and the pulley (Fig. 2.1).

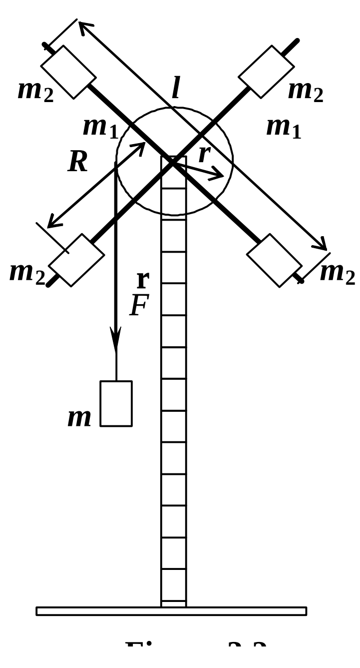
3 DESCRIPTION OF EXPERIMENTAL APPARATUS

Oberbek pendulum shown in Fig. 3 is a cross-piece which consists of two rods, of each of mass m_1, and weights of identical mass m_2, mounted on the rods on distance R from the axis of rotation. The pendulum axis of rotation passes through its center and center of sheave, which a string is coiled round. On the string a weight of mass m is suspended.

Under action of the weight m the coiled string unwinds and a pendulum start to rotate with constant angular acceleration. Position of the weight m is read from a vertical scale.

4 DERIVATION OF COMPUTATION FORMULAS

The moment of inertia of Oberbek pendulum can be found by two methods. The first is based on application of formula (2.4) to the system under consideration. The second



method consists in the use of the fundamental law of dynamics of rotational motion of a rigid body, where quantities *M* and *e* are expressed through experiment measurables.

The first method.

Being an additive quantity, the moment of inertia of Oberbek pendulum equals to the sum of moments of inertia of cross-piece, four weights on rods, and sheave. Moment of inertia of cross-piece (two rods) about its center is equal

$$I_x = 2\frac{1}{12}m_1 l^2 = \frac{1}{6}m_1 l^2, \tag{4.1}$$

(here we make use of formula (2.8)). Moment of inertia of system of four weights equals

$$I_в = 4m_2 R^2 \tag{4.2}$$

(a formula (2.6) has been used). Application of formula (2.6) for calculation of total moment of inertia of weights is justifiable when condition $R^2 >> a^2$ is fulfilled (R being the distance from the axis of rotation to the center of masses of the weight, a being linear dimension of weight); in this case the weight can be treated as a particle (material point).

At conditions

$$m_1 + 4m_2 >> m_{sh}, \tag{4.3}$$

$$l >> r, \qquad R >> r \tag{4.4}$$

(m_{sh} is mass of the sheave), one may neglect the moment of inertia of sheave. Finally, the total moment of inertia equals

$$I = \frac{1}{6}m_1 l^2 + 4m_2 R^2. \tag{4.5}$$

The second method.

Moment of inertia of pendulum can be found also from the fundamental law of dynamics of rotational motion of a rigid body (2.11).

In this experiment the tension force of string which sets a cross-piece in motion. On the basis of the second law of Newton, at weight descending, this force equals

$$F=mg-ma=m(g-a) \tag{4.6}$$

where g is the free fall acceleration; a is acceleration of suspended weight.

The lever arm of force F is equal to the radius of sheave r, so the torque is

$$M=Fr=m(g-a)r. \tag{4.7}$$

Angular acceleration ε can be calculated if distance h, passed by weight m, and time of descending are known. It is easy to obtain from kinematical equations, that

$$h=at^2/2, \tag{4.8}$$

therefore

$$a=2h/t^2. \tag{4.9}$$

Consequently, as (on a basis of equation (2.3))

$$\varepsilon=a/r, \tag{4.10}$$

we obtain

$$\varepsilon=2h/(t^2 r). \tag{4.11}$$

Substituting equations (4.7), (4.10), (4.11) into formula (2.11), after replacement r=d/2 (d is diameter of sheave), we get a computation formula for determination of Oberbek pendulum moment of inertia :

$$I = \frac{md^2}{4}\left(\frac{gt^2}{2h} - 1\right). \tag{4.12}$$