Student's name ____

Experiment № 2

STUDY OF TRANSLATIONAL MOTION LAWS WITH ATWOOD MACHINE

Objective of the experiment: Study of the translation motion laws for a rigid body. Determination of translation motion acceleration.

1 EQUIPMENT

1. Experimental apparatus (Atwood machine).
2. Two identical weights bound together by a cord.
3. Set of different mass rings.

2 THEORY

2.1 There are three approaches, equally appropriate for description of a particle (point-like object) motion. These are trajectory, position-vector and coordinates approaches. The trajectory approach is the most convenient one, if a trajectory is known. Trajectory is a line, along which the point moves. In the position vector (radius-vector) approach, the position of a point at every moment is determined by a vector $\vec{r}$, whose beginning is superposed with the origin of reference frame chosen and end is superposed with the point. One obtains a coordinate description by decomposition of position vector in three coordinate components, each being a vector directed along one of the coordinate axes:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}, \tag{2.1}$$

where x, y, z are projections of position-vector $\vec{r}$ on the respective coordinate axes of Cartesian coordinate system; $\vec{i}$, $\vec{j}$ and $\vec{k}$ are unit vectors directed along the respective axes.

2.2 Instantaneous velocity of a particle may be defined as

$$\vec{J} = \frac{d\vec{r}}{dt} = J_x\vec{i} + J_y\vec{j} + J_z\vec{k}, \tag{2.2}$$

where J_x, J_y, J_z are the projections of velocity vector $\vec{J}$ on coordinate axes:

$$J_x = \frac{dx}{dt}, \qquad J_y = \frac{dy}{dt}, \qquad J_z = \frac{dz}{dt}. \tag{2.3}$$

The unit of velocity in metric system is one meter per second, 1 $\frac{m}{s}$. The velocity of a body's motion with respect to inertial frame of reference can be changed under the external influences (forces exerted on the body). This change is characterized by an acceleration, which is defined as time rate of velocity change:

$$\vec{a} = \frac{d\vec{J}}{dt} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}, \tag{2.4}$$

here, a_x, a_y, a_z are projections of the acceleration vector on coordinate axes:

$$a_x = \frac{dJ_x}{dt}, \qquad a_y = \frac{dJ_y}{dt}, \qquad a_z = \frac{dJ_z}{dt}. \tag{2.5}$$

The unit of acceleration in metric system is one meter per second square, 1 $\frac{m}{s^2}$.

2.3 Let us consider kinematic relations for uniform and accelerated motions.

a) Uniform motion is a motion with constant velocity.

If motion is rectilinear (one-dimensional), it is convenient to take advantage of coordinate description and direct the OX -axis along the direction of body's motion):

$$J_x = \frac{dx}{dt}, \qquad a_x = \frac{dJ_x}{dt}, \tag{2.6}$$

all other components of velocity and acceleration equal zero. As $J_x = const$ at uniform motion, from the latter equation we immediately obtain $a_x = 0$, and integration of the equations (2.6) with respect to time gives

$$x = x_0 + J_x t, \tag{2.7}$$

where x denotes the position of moving particle at the moment of time t, and x_0 denotes the position of particle at the initial moment ($t=0$).

b) Uniformly accelerated motion (acceleration is constant $a_x = const$).

If a motion is one-dimensional (along OX -axis), we have $a_{\mathrm{x}} = \mathrm{const}$, successive integration of formulas (2.6) provides us with the expressions for the particle velocity as a function of time

$$J_x = J_{0x} + a_x t, \tag{2.8}$$

and the particle position as a function of time

$$x = x_0 + J_{0x} t + \frac{a_x t^2}{2}; \tag{2.9}$$

here x_0 stands for initial position of the particle (at the moment $t=0$), J_{0x} denotes initial velocity along OX -axis. These equations constitute the system of kinematic equations for accelerated motion.

The value of velocity increases in magnitude if projections J_x and a_x have the same sign, and vice versa.

2.4 Dynamics studies the laws of mechanical motion by revealing the causes of this motion.

In dynamics the notions of force and mass are fundamental. Interaction of bodies may cause either deformation of body or acceleration of it. Force is the measure of this interaction. Force is a quantity, which is characterized by the point of application, a magnitude and a direction of the action. If a few forces $\vec{F}_1, \ \vec{F}_2, \ \ldots \ \vec{F}_n$ are exerted on a body at the same time, a total (resultant) force $\vec{F}$ is found by the superposition principle: $\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n$.

Inert mass is the measure of a body's inertia. Inert mass m quantitatively characterizes the matter, which a body consists of. Inertia means the property of a matter to resist the change of the state of motion.

Three laws formulated by Isaac Newton are fundamental in dynamics.

<u>The first Newton's law is:</u> t<u>here are reference frames (inertial RF) in which a body remains in rest or uniform translational motion if no external force act on it or external forces balance each other</u>. This law is called the law of inertia as well.

In the inertial frames of reference at velocities J, considerably less than velocity of light $(J^2 << c^2)$ <u>the second Newton's law</u> takes place:

$$\vec{F} = m\vec{a} \qquad \text{or} \qquad \vec{a} = \frac{\vec{F}}{m} \tag{2.10}$$

that is, acceleration a body moves with, is proportional to resultant of all forces, exerted on the body, and inversely proportional to mass of the body.

Equation (2.10) sets up a definition for unit of force, newton (N): 1 N is the force which provides the body of 1 kg mass with acceleration of 1 m/s^2 (1 N = 1 kg·m/s^2).

The third Newton's law: If a body A acts on a body B with force $\vec{F}_1$, then body B acts on a body A with force $\vec{F}_2$, these forces being equal in magnitude and opposite in direction:

$$\vec{F}_1 = -\vec{F}_2. \tag{2.11}$$

The third Newton's law states the equality of action and counteraction.

2.5 The simplest motions of a rigid body are translational motion and rotational motion around a fixed axis. In translational motion all points of a body move with identical velocities and accelerations along identical trajectories. Any line, that passes through a body, moves parallel to itself. In rotational motion of a rigid body around fixed axis all points of the body move on circular trajectories, the centers lying on the same line called the rotation axis. Circles, the points of body move on, lie in the planes perpendicular to the axis of rotation. Points of the body, lying on the rotation axis are immobile.

At translational motion velocities and accelerations are the same for all points of a rigid body, so the kinematic relations, derived for a particle, can be applied to describe the motion of body's center of mass. The mass center of the system of bodies moves as a particle with mass, equal to the total mass of the system, to which resultant force is applied. Rasultant force is equal to the vector sum of all acting forces.

3 DESCRIPTION OF EXPERIMENTAL APPARATUS

The experimental apparatus (Fig. 3.1), known as Atwood machine, consists of pulley B, over which two identical weights C and C′ with masses m bound together by a cord are suspended. The pulley B is mounted on a stand A provided with a millimeter scale. On the stand an immobile bracket E with an electromagnet and mobile brackets F and G with built-in photogates are mounted, too. If ring D of mass m_1 is placed onto the weight C, the bound weights start accelerated motion. An electronic block is turned on by pressing the "ON/OFF" button. In an initial state electromagnet E holds the weight C′ in the lowest position. The ring D should be placed onto the weight C. The reading of electronic block should then be cleared off by pressing the "CLEAR" button. To start an experiment, the button "START" is to be pressed, then an electromagnet is turned off and the weights motion starts. The weight C moves with constant acceleration from its upper position to the bracket F. The ring D being taken off from the weight C by the bracket F, the weights C and C′ move uniformly. The photogates start measuring the time at the moment when the weight C is passing the bracket F and stop the measurement when the bracket G is reached. From the display of the electronic block the time of uniform motion between points F and G

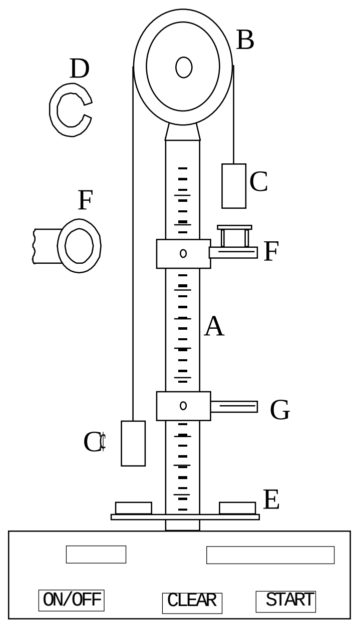

Figure 3.1

can be read from. Distance between these brackets is measured by the millimeter scale on the stand.

4 DERIVATION OF COMPUTATION FORMULAS

4.1 In order to derive the system acceleration one may apply the system of kinematic equations for accelerated motion. Initial velocity of the weights equals zero. In time interval t_1 the weight C, moving with acceleration, passes the path

$$S_1 = \frac{a_1 \cdot t_1^{\ 2}}{2}, \tag{4.1}$$

its speed reaches the value

$$J = a_1 \cdot t_1. \tag{4.2}$$

After the ring D is taken off while passing the bracket F, the weight C moving with constant velocity, covers a distance

$$S_2 = J \cdot t_2 \tag{4.3}$$

in time t_2.

From equations (4.1)-(4.3) we obtain the for the acceleration:

$$a_1 = \frac{S_2^2}{2S_1 t_2^2}. \tag{4.5}$$

4.2 Acceleration of the system can be determined using the laws of dynamics as well.

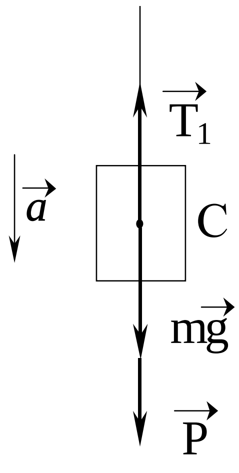

Figure 4.1

The weight C is under the actions of the gravitation field of Earth, the cord it is suspended on, the ring D and surrounding air. Neglecting the friction in air, the second law of Newton for the weight C can be written down as follows:

$$m\vec{g} + \vec{P} + \vec{T}_1 = m\vec{a}_1, \tag{4.6}$$

where m is the mass of the weight C, $\vec{g}$ is a free fall acceleration, $\vec{P}$ is the force, exerted by the ring D on the weight C, $\vec{T}_1$ is tension force of the string. Projecting equation (4.6) on the vertical axis we obtain:

$$T_1 - mg - P = -ma_1. \tag{4.7}$$

Analogously, one obtains the equation of motion for the weight C′:

$$T_2 - mg = ma_2, \tag{4.8}$$

and the equation of motion for the ring D:

$$N - m_1 g = -m_1 a_1, \tag{4.9}$$

here N stands for a magnitude of the reaction force, acting on the ring D from the weight C. It follows from the third Newton's law that $N=P$. As the cord is considered to be untensile, acceleration magnitudes for all the weights are equal $a_1 = a_2 = a$. The cord tension forces can be considered equal only if the pulley moment of inertia may be neglected and a friction is absent. If mass of the pulley and a friction are neglected, then $T_1=T_2=T$. From equations (4.7)-(4.9) one may obtain the expression for the acceleration:

$$a = \frac{m_1}{2m + m_1} g. \tag{4.10}$$

Expressions (4.5) and (4.10) are the computation formulas of the experiment.

5 PROCEDURE AND ANALYSIS

5.1 Mount brackets G and F on a distance S_2 specified by laboratory assistant

5.2 Put the weight C′ in the lowest position. Turn on an electronic block. Place the ring D onto the weight C (weight of ring D is specified by lab assistant).

5.3 Note initial position of the weight C on a millimeter scale and determine the distance S_1.

5.4 Press the button “START” to put the system in motion and measure time t_2.

5.5 Repeat the experiment two times more. Determine the average values of S_1 and t_2.

5.6 Calculate acceleration using formula (4.5).

5.7 Determine masses of the weights used.

5.8 Calculate acceleration using formula (4.10).

5.9 Compare the values of the acceleration calculated using formulas (4.5) and (4.10).

5.10 Estimate errors of measurements and calculations.

5.11 Express results of the calculation in a form $a = a_m \pm \Delta a_m$ and specify the value of relative error ε.

5.12 Fill the tables 5.1 and 5.2 with results of experiments and calculations.

Table 5.1

N	S_1 10^{-3} m	ΔS_1 10^{-3} m	S_2 10^{-3} m	ΔS_2 10^{-3} m	t_2 s	Δt_2 s	a, m/s^2	Δa m/s^2	ε %
1									
2									
3									
Mean value									

Table 5.2

m 10^{-3} kg	Δm 10^{-3} kg	m_1 10^{-3} kg	Δm_1 10^{-3} kg	g m/s^2	Δg m/s^2	a, m/s^2	Δa m/s^2	ε %