Experiment M 1
DETERMINATION OF DENSITY OF BODY OF REGULAR GEOMETRICAL SHAPE

Objective of the experiment: To master a practical method of substance density determination. To practice a method of errors estimation.

1 EQUIPMENT

1. Body of regular geometrical shape (regular cylinder).
2. Vernier caliper.
3. Micrometer.
4. Scales.

2 THEORY

2.1 Mass of a body is a physical quantity which measures body's inertia in translational motion. Inertia is a property of body to maintain its state of motion or rest if no external force acts on it or external forces compensate each other. Mass is one of basic properties, it depends on the size of the body and properties of matter, which the body consists of. Mass can characterize not only inert properties of matter but also its gravitational ability to attract other bodies.

The magnitude of mass can be found using different manifestations of it (inertia, gravitational interaction), by comparison with mass of standard body, by convention taken as unit. In International System of units (SI) a kilogram is the unit of mass (1 kg). Mass of body, as a rule, is determined by weighing on scales.

2.2 Density r of homogeneous body of mass m and volume V is given by a formula:

$$r = \frac{m}{V} . \tag{2.1}$$

One can see that density is a physical quantity, which magnitude is equal to mass of volume unit of homogeneous matter. In formula (2.1) all quantities must be expressed in the same system of units.

By the same formula, average density of heterogeneous matter is given:

$$\langle r \rangle = \frac{m}{V} . \tag{2.2}$$

In agreement with equations (2.1) and (2.2), mass of homogeneous body can be expressed as

$$m = r \cdot V , \tag{2.3}$$

and mass of heterogeneous body as

$$m = \langle r \rangle \cdot V . \tag{2.4}$$

In International System of units (SI) the unit of density is one kilogram per cube meter ($[r] = 1 \frac{kg}{m^3}$). For the heterogeneous matter the density in some point is defined as a limit of

relation of mass Δm of some amount of matter in the vicinity of the given point to its volume ΔV on condition that a volume is infinitesimally small. One should remember that the value of density depends on a temperature (in tables for substances density values this temperature is specified also).

2.3 For determination of volume of bodies of arbitrary geometrical shape, the method of the hydrostatical weighing is used among others. If we have the body of regular geometrical shape, its volume can be calculated by known formulas, after its linear dimensions having been measured. For precise measurements of linear dimensions the micrometers and vernier calipers are used.

3 DERIVATION OF COMPUTATION FORMULAS

a 3.1 Volume of cylinder can be calculated by formula:

$$V = \frac{pd^2}{4} h, \tag{3.1}$$

where d is diameter of cylinder, h is its height. From formula (2.1) we get a computation formula for calculation of density:

$$r = \frac{4m}{p \cdot d^2 \cdot h}. \tag{3.2}$$

For determination of density it is necessary to carry out the precise measurement of the following quantities: mass (using scales), diameter (using micrometer or vernier caliper) and height of cylinder (using vernier caliper).

To improve accuracy, every direct measurement is carried out a few times (series of n measements for each of quanties m, d, h).

For computation of density by formula (3.2) it is necessary to find the average value of each of quantities m, d, h. We calculate the mean value of mass by formula:

$$\langle m \rangle = \frac{1}{n} \sum_{i=1}^{n} m_i, \tag{3.3}$$

where m_i is the value of mass, found in the ith measurement (i runs over values from 1 to n). In the same way we find the average values of d and h.

3.2 Random errors of the direct measurements can be calculated by the method described below.

Method of average deviation.

Each of the measurements of mass is characterized by an absolute error

$$\Delta m_i = m_c - m_i. \tag{3.4}$$

The average value of deviation is characterized by an mean absolute error

$$\langle \Delta m \rangle = \frac{1}{n} \sum_{i=1}^{n} |\Delta m_i|. \tag{3.5}$$

3.2 The overall error of the direct measurement is determined by adding random error term Dm_v and instrumental errors Δm_i (from specification of instruments):

$$\Delta m_n = \sqrt{\langle \Delta m \rangle^2 + (\Delta m_{instr})^2}. \tag{3.7}$$

The errors of other direct measurements are found in the similar way.

3.3 Following method is applied for estimation the error of the density calculated by formula (3.2). A relative error is found from formula:

$$e = \frac{\Delta r}{r} = \frac{\Delta m}{m} + 2\frac{\Delta d}{d} + \frac{\Delta h}{h} , \qquad (3.9)$$

and absolute error can be calculated from relation $\Delta r = e \cdot r$. The result of experiment is given in a form, which specifies the value of relative error e also.

4 PROCEDURE AND ANALYSIS

4.1 Using analytical scales measure the mass of studied cylinder n times (the number n is specified by lab assistant).

4.2 Using micrometer measure the diameter of the cylinder n times.

4.3 Using vernier caliper measure the height of cylinder n times.

4.4 Calculate the mean values of the measured quantities and errors of measurements and calculations.

4.5 Fill in the table 4.1 with the results of measurements and calculations.

Table 4.1

N	d 10^{-3} m	Dd 10^{-3} m	m 10^{-3} kg	Dm 10^{-3} kg	H 10^{-3} m	Dh 10^{-3} m	r kg/m^3	Dr kg/m^3	e %
1									
...									
n									
Mean value									

4.6 Express results of the calculation of a form $m = \langle m \rangle \pm \langle \Delta m \rangle$ and specify the value of relative error ε.

5 CONTROL QUESTIONS

5.1 What is a mass of body?

5.2 What is a density of body?

5.3 Estimate average density of physics textbook.

Mean value of the cylinder's diameter

$$\langle d \rangle = \frac{d_1 + d_2 + d_3}{3} =$$

Indeterminate errors of the cylinder's diameter measurements

$$\Delta d_1 = \langle d \rangle - d_1 =$$

$$\Delta d_2 = \langle d \rangle - d_2 =$$

$$\Delta d_3 = \langle d \rangle - d_3 =$$

Average deviation of the cylinder's diameter

$$\langle \Delta d \rangle = \frac{|\Delta d_1| + |\Delta d_2| + |\Delta d_3|}{3} =$$

Overall deviation

$$\Delta d = \sqrt{\langle \Delta d \rangle^2 + (\Delta d_{instr})^2} =$$

Mean value of the cylinder's mass

$$\langle m \rangle = \frac{m_1 + m_2 + m_3}{3} =$$

Indeterminate errors of the cylinder's diameter measurements

$$\Delta m_1 = \langle m \rangle - m_1 =$$

$$\Delta m_2 = \langle m \rangle - m_2 =$$

$$\Delta m_3 = \langle m \rangle - m_3 =$$

Average deviation of the cylinder's mass

$$\langle \Delta m \rangle = \frac{|\Delta m_1| + |\Delta m_2| + |\Delta m_3|}{3} =$$

Overall deviation

$$\Delta m = \sqrt{\langle \Delta m \rangle^2 + (\Delta m_{instr})^2} =$$

Mean value of the cylinder's height

$$\langle h \rangle = \frac{h_1 + h_2 + h_3}{3} =$$

Indeterminate errors of the cylinder's diameter measurements

$$\Delta h_1 = \langle h \rangle - h_1 =$$

$$\Delta h_2 = \langle h \rangle - h_2 =$$

$$\Delta h_3 = \langle h \rangle - h_3 =$$

Average deviation of the cylinder's diameter

$$\langle \Delta h \rangle = \frac{|\Delta h_1| + |\Delta h_2| + |\Delta h_3|}{3} =$$

Overall deviation

$$\Delta h = \sqrt{\langle \Delta h \rangle^2 + (\Delta h_{instr})^2} =$$

Density of the cylinder:

$$r = \frac{4m}{p \cdot d^2 \cdot h} =$$

Relative error

$$e = \frac{\Delta r}{r} = \frac{\Delta m}{m} + 2\frac{\Delta d}{d} + \frac{\Delta h}{h} =$$

Absolute error

$$\Delta r = e \cdot r =$$

The final result for cylinder's density

$$r = \left(\qquad \pm \qquad \right)\frac{kg}{m^3}$$