

Wave and quantum optics

Chapters 1-5 from the textbook

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1. Classical Optics. Ray and wave models of optics

1.1 Optics. Dual nature of light. Models of light

Optics is a part of physics which studies the light nature, light phenomena, interaction between light and matter.

Light must be regarded as having a dual nature:

- 1) It exhibits the characteristics of a wave in some situations (interference, diffraction, polarization) and
- 2) It exhibits the characteristics of a particle in other situations (photoelectric effect, absorption and radiation spectra).

The question “Is light a wave or a particle?” is inappropriate. Sometimes light acts like a wave, and other times it acts like a particle.

The nature and properties of light have been a subject of great interest since ancient times. The Greeks believed that light rays consist of tiny particles (corpuscles) that are emitted by a light source, moves straightforwardly and that these particles stimulated the perception of vision upon striking the observer’s eye. Newton used this particle theory to explain the reflection and refraction (bending) of light. In 1678, one of Newton’s contemporaries, Christian Huygens was able to explain many other properties of light by proposing that light is a wave. In 1801, Thomas Young showed that light beams can interfere with one another, giving strong support to the wave theory. In 1865, Maxwell created the theory of electromagnetic field which treated visible light as electromagnetic waves of some spectral interval). The basics of Maxwell’s theory have been explained in the previous part of the course.

Although the wave model and the classical theory of electricity and magnetism were able to explain most known properties of light, they could not explain subsequent experiments as **the thermal radiation emitted by hot objects, the photoelectric effect** and others.

At the beginning of the twentieth century, Max Planck returned to the particle theory of light to explain **the radiation emitted by hot objects**. Einstein then used the particle theory to treat the **photoelectric effect**, discovered by Hertz, and to show how electrons are emitted by a metal exposed to light. The quantization model assumes that the energy of a light wave is present in particles called **photons**; hence, the energy is said to be quantized. The particle model is used in order to explain the interaction of light with individual atoms. According to Einstein’s theory, the energy of a photon is proportional to the frequency of the electromagnetic wave: $\epsilon = \hbar\omega$

($\hbar = \frac{h}{2\pi}$), where $h=6,63\cdot 10^{-34}$ J·s – Planck’s constant. Photons have mass $m_f = \frac{\hbar\omega}{c^2}$ and linear

moment $p_f = \frac{\hbar\omega}{c}$. The angular frequency of wave is $\omega = \frac{2\pi c}{\lambda}$, where c is the speed of light and

λ is the wavelength.

The wave model is used to explain the interaction of light with material objects of sizes comparable to or smaller than a wavelength of light. Waves of the visible light have wavelength λ belonging to the region 0.38- 0.76 micrometers roughly.

If wavelength $\lambda \rightarrow 0$ the ray model of light is used, which exploits the experimental fact that in homogenous media the light travels along the straight line. The ray model of light seems natural when we observe such phenomena like sunbeams coming through holes in clouds. Having already been introduced to the concept of light as an electromagnetic wave, you know that the ray model is not the most general theory of light, but the ray model is simpler, and in many cases it is sufficient for an adequate description of natural phenomena.

Therefore, there are three models of light: ray model, wave model and particle model each applicable to particular class of phenomena.

1.2 The ray optics laws

The ray model of light is useful when light is interacting with material objects that are much larger than a wavelength of light. Since a wavelength of visible light is so short compared to the human scale of existence, the ray model is useful in many practical cases.

When light reaches the boundary between two different substances, a part of the light is reflected (bounces back) and a part passes on into the new medium. It is typical behaviour for a wave (light wave, in our case). The phenomenon of reflection occurs only at the boundary between two media, just like the change in the speed of light that passes from one medium to another. The part of incident light energy is transmitted from the first medium to the second one (see fig. 1.1) and forms the transmitted (refracted) ray, which is deviated from the original direction of travel.

There is a difference between reflections from smooth and rough surfaces. A smooth surface produces the specular reflection, in which the reflected ray exits at the same angle with respect to the normal as that of the incident ray. A rough surface gives diffuse reflection, where a single ray of light is divided up into many weaker reflected rays going in many directions.

As result of many observations the ray optics laws were formulated:

1. The incident (1), reflected (1'), transmitted (1'') rays and normal to the surface in the point of incidence lie in the same plane (fig. 1.1)

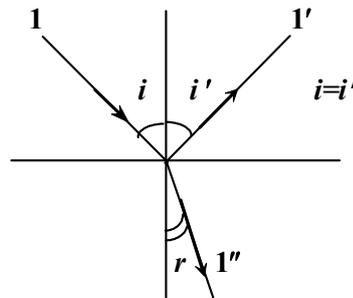


Fig.1.1

2. The angle of the reflection is the same as that of the incidence ($i = i'$).

3. The incident angle i and refracted angle r are related by **Snell's law**: $\frac{\sin i}{\sin r} = \frac{g_1}{g_2} = n_{12}$. Here

g_1 is the light velocity in first medium (for the incident ray), g_2 is the light velocity in second medium (for the refracted ray), n_{12} is relative index of refraction for the two media.

If the first medium is a vacuum, the relation takes the form $\frac{\sin i}{\sin r} = n = \frac{c}{g}$. Coefficient n in this

case is called absolute refraction index of medium (refraction index with respect to vacuum). Here c is velocity of light in vacuum, g is velocity of light in refractive medium.

The optical properties of a medium are determined by its electric properties. Absolute refraction index $n \approx \sqrt{\epsilon}$, where ϵ is the electric permeability of the medium.

Remember! 1. Light energy can also be absorbed by matter, i.e., converted into heat.

2. As light travels from one medium to another, its frequency does not change but its wavelength does. The wavelength of light λ_n in a medium whose index of refraction is n changes as $\lambda_n = \lambda/n$.

Any wave is partially transmitted and partially reflected at the boundary between two media in which velocities of light are different.

Important technologies such as fiber optics are based on the fact that this fraction becomes zero for sufficiently oblique angles. This phenomenon is referred to as total internal reflection. It occurs when there is no refracted angle that satisfies Snell's law.

As we see from fig.1.2 total internal reflection will be observed, if $\sin i_{tot} = \frac{1}{n}$ and

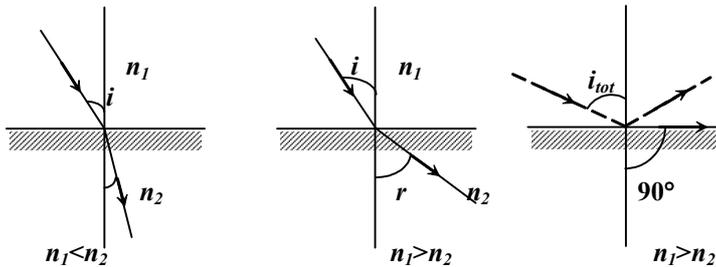


Fig.1.2

$$\sin i_{tot} = n_{21}$$

Here $i_{tot.ref}$ is critical angle of total internal reflection. This phenomenon can occur only if light travels from a medium of high index of refraction to one of lower index of refraction.

The Snell's law can be derived from the principle of the least time for light motion, which states:

Of all the conceivable paths light could follow, the only one that is physically possible is the path that takes the least time.

The principle of the least time works not only for light in a vacuum and light undergoing reflection. It works for the bending of light when it passes from one medium into another as well. For practical application of this principle, in optics, an optical path is introduced besides geometrical path.

If geometrical path is $\Delta l = l_2 - l_1$, the optical path $\Delta L = n\Delta l = n(l_2 - l_1)$. Optical path accounts the difference between light velocity in various media and allows to use the rules of geometry in optical calculations.

2. Interference

2.1 Conditions of interference

Interference is the phenomenon of coherent waves combination in some region of space which results in enhancement or reducing of the oscillation in this region.

The waves must be **coherent**, which means that

- 1) the frequencies of waves are equal;
- 2) the waves must maintain a constant phase with respect to each other and the waves must have identical wavelengths;
- 3) directions of oscillations produced by these waves are the same.

If equations for the electric field vector projections for two electromagnetic (light) waves are

$$E_1 = E_{m_1} \cos(\omega t - kx),$$

$$E_2 = E_{m_2} \cos(\omega t - kx + \varphi_2),$$

then the phase with respect to each other is $\Delta\varphi = \varphi_2$ and the waves are coherent, because the phase shift in every point is constant and frequencies are equal. Such waves can produce constructive (maximal amplitude of resultant oscillation) or destructive (minimal amplitude of resultant oscillation) interference.

If two ordinary light sources are placed side by side, however, no interference effects are observed, because the light waves from one source are emitted independently of the waves from

the other source. Hence, the emissions from the two sources don't maintain a constant phase relationship with each other during the time of observation. An ordinary light source undergoes random phase changes about once every 10^{-8} s. Therefore, constructive interference, destructive interference, and intermediate states are stable only during time intervals of the order of 10^{-8} s. The result is that no interference effects are observed, because the human eye can't follow such short-term changes. Ordinary light sources are said to be **incoherent**.

An older method for producing two coherent light sources is to pass light from a single wavelength (monochromatic) source through a narrow slit and then allow the light to fall on a screen containing two other narrow slits (Fig.2.1) The first slit is needed to create a single wave-front that illuminates both slits coherently. The light emerging from the two slits is coherent because a single source produces the original light beam and the slits serve only to split the original beam into two parts. Any random change in the light emitted by the source will occur in the two separated beams at the same time, and interference effects can be observed.

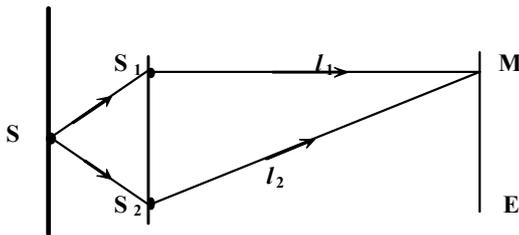


Fig .2.1

Currently it's much more common to use a laser as a coherent source to demonstrate interference. As is shown on fig 2.1, light is incident on a screen containing a narrow slit S. The light waves emerging from this slit arrive at a second screen that contains two narrow, parallel slits S_1 and S_2 . These slits serve as a pair of coherent light sources because waves emerging from them originate from the same wave front and therefore are always in

phase. The light from the two slits produces a visible pattern on screen E consisting of a series of bright and dark parallel bands called **fringes** .When the light from slits S_1 and S_2 arrives at a point on the screen so that constructive interference occurs at that location, a bright fringe appears. When the light from the two slits combines destructively at a location on the screen, a dark fringe results. The light intensity on the screen at some point is the result of light interference from both slits. Depending on the wavelength λ and the distances l_1 and l_2 passed by light from sources S_1 and S_2 , respectively, in the particular point M at the screen E the constructive or destructive interference can be observed.

If the distance $\Delta=l_2 - l_1$, known as path difference, fulfils the condition

$$\Delta = (l_2 - l_1) = m\lambda ,$$

where $m = 0, 1, 2, 3, \dots$ and λ is wavelength, the maximum of light intensity is observed at point M.

If the path difference Δ fulfils the condition $\Delta = (2m + 1)\frac{\lambda}{2}$ then in point M the minimum of light intensity is observed.

2.2 Interference on a thin film

Interference effects can be observed in thin films, such as thin layers of oil on water surface or the thin surface of a soap bubble. The varied colours observed when white light is incident on such films result from the interference of waves reflected from the two surfaces of the film.

Let us consider a thin film with parallel upper and bottom surfaces. Assume that the light rays travelling in air are normal to the two surfaces of the film. To determine whether the reflected rays interfere constructively or destructively, we first note the following facts:

1) A wave travelling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change upon reflection when $n_2 > n_1$ and undergoes no phase change if $n_2 < n_1$.

2) The wavelength of light λ_n in a medium whose index of refraction is n is determined by formula $\lambda_n = \lambda/n$ (λ_n is the wavelength of the light in free space. The difference between the refraction indices of air and free space is negligible, $n_{\text{air}} \approx 1$).

Let us apply these facts to the film where index refraction $n_{\text{film}} > 1$ and film thickness is d . On fig 2.2 interference between the ray reflected from top surface 1 and bottom surface 2 is shown.

From fig. 2.2.a one can see that the wavelength λ_0 of the incident light is larger than the wavelength λ of the light propagating in medium, because the speed of light in vacuum is always greater than that in medium. In fig. 2.2.b the wave reflected from the upper surface of film is shown. The ray reflected from the upper surface undergoes a phase change of 180° with respect to the incident wave.

The ray reflected from the lower film surface (fig. 2.2.c) undergoes no phase change because it is reflected from a medium (air) that has a lower index of refraction. Therefore, reflected rays are 180° out of phase. It is equivalent to a path difference of $\lambda/2$. However, we must also consider that ray in fig. 2.2.c travels a distance $2d$ before the waves recombine in the air above upper surface. If $2d > \lambda/2$, then reflected rays recombine in phase, and the result is constructive interference.

In general, the condition for constructive interference in thin films is $2d = (m + 1/2)\lambda$ ($m = 0, 1, 2, 3, \dots$)

If the extra distance $2d$ travelled by ray 2 corresponds to a multiple of λ , then the two waves combine out of phase, and the result is destructive interference which is the case illustrated by figure 2.2 (total intensity if the reflected light is zero).

The general equation for destructive interference in thin films is $2nt = m\lambda$ ($m = 0, 1, 2, 3, \dots$).

The phenomenon of interference has many applications in science and technics.

Glass lenses used in cameras and other optical instruments are usually coated with a transparent thin film to reduce or eliminate unwanted reflection and enhance the transmission of light through the lenses due the phenomena of interference on thin film. This method has been invented by prominent Ukrainian physicist Olexandr Smakula.

Very important modern tools for precise measurements are interferometers. These are the devices, which can be used to measure wavelengths or other lengths with unsurpassed accuracy and the phenomenon of interference.

For example, a hologram is the stable interference pattern created by light sources (lasers, as a rule). It is used by engineers for studies of deformations under pressure, by biologists for studies of the structure of bacteria. Artists use of holograms in concerts. Even banks use holograms to protect documents from falsification.

Compact disks (CD's) and digital video disks (DVD's) have revolutionized the computer and entertainment industries by providing fast access; high-density storage of text, graphics, and movies; and high-quality sound recordings. The data on these disks are stored digitally as a series of zeros and ones, and these zeros and ones are read by laser light reflected from the disk. A CD is read by focusing a near infrared semiconductor laser through the bottom of the polycarbonate layer. The change in height between pits and lands results in a difference in the way the light passes (denoted d in fig. ...). Due to interference of light reflected from the upper

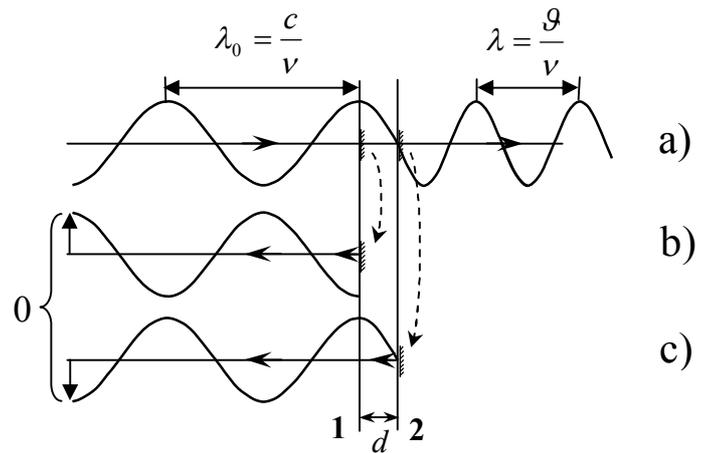


Fig. 2.2

and the bottom sides of the polycarbonate layer maxima and minima are produced. By measuring the intensity change with a photodiode, the data can be read from the disc.

3. Diffraction

If the wave meets a barrier in which there is a circular opening whose diameter is much larger than the wavelength, the wave emerging from the opening continues to move along a straight line (apart from some small edge effects); hence, the ray approximation is valid. If the diameter of the opening is on the order of the wavelength, the waves spread out from the opening in all directions. This effect is called *diffraction*. Such behavior indicates that light, once it has passed through a narrow slit, spreads beyond the narrow path defined by the slit into regions that would be in shadow if light traveled in straight lines. Other waves, such as sound waves and water waves, also have this property of spreading when passing through apertures or by sharp edges.

Diffraction can be defined in the following way:

Diffraction is the deviation of light from a straight-line path when the light passes through an aperture or around an obstacle.

There are two types of diffraction: Fresnel's diffraction and Fraunhofer diffraction. Type of diffraction depends on distance between source of light and object of diffraction.

3.1 Fresnel's diffraction on a disk

When an opaque object is placed between a point source of light and a screen, no sharp boundary exists on the screen between a shadowed region and an illuminated region. The illuminated region above the shadow of the object contains alternating light and dark fringes. Such a display is called a diffraction pattern. Diffraction pattern associated with the shadow of a small disk (for example small coin) is an example of Fresnel's diffraction (fig. 3.1). A bright spot occurs at the center, and circular fringes extend outward from the shadow's edge. From the viewpoint of geometrical optics (in which light is viewed as rays travelling in straight lines), we expect the center of the shadow to be dark because that part of the viewing screen is completely shielded by the coin. We can explain the central bright spot only by using the wave theory of light, which predicts constructive interference at this point.

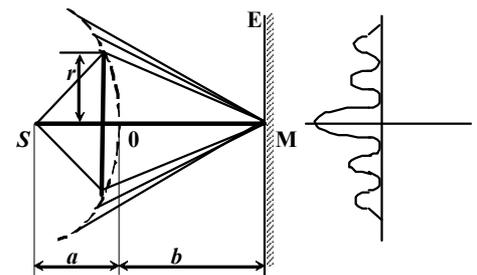


Fig. 3.1

Short explanation of this fact can be given on the basis of Huygens-Fresnel's principle. It states that every point reached by the wave becomes a source of secondary wave and these waves interfere to produce the light pattern. All points in the wave front can be divided in several symmetrical zones (fig. 3.2) with the following properties:

1. The edge points of every zone are shifted by distance $\lambda/2$ from neighbouring zone. This means that the neighbouring zones produce oscillations with opposite phases and thus interfere destructively at the image point.
2. Amplitude and phase of oscillations which come to the point from light wave front depend on number m of zones in front. If in the center of the wave front an

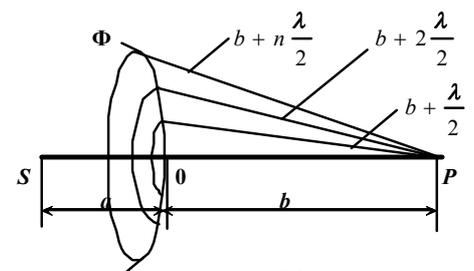


Fig. 3.2

obstacle is placed, it closes m of Fresnel zones, so electromagnetic radiation of these zones do not contribute to the total light amplitude A . This means that if m zones are closed by disk, in image point one has $A = A_{m+1}/2 > 0$. And as result bright spot occurs at the center of screen .

3.2 Fraunhofer's diffraction on a single slit

Let us consider a common situation, that of light passing through a narrow opening modelled as a slit, and projected onto a screen. To simplify our analysis, we assume that the observing screen is far from the slit, so that the rays reaching the screen are approximately parallel. This can also be achieved experimentally by using a converging lens to focus the parallel rays on a nearby screen. In this model, the pattern on the screen is called Fraunhofer's diffraction pattern.

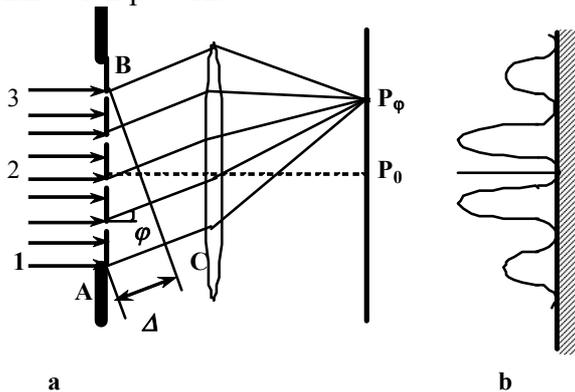


Fig 3.3

Figure 3.3 a shows light entering a single slit and diffracting as it propagates toward a screen.

The Fraunhofer diffraction pattern produced by a single slit of width a on a distant screen consists of a central bright fringe with intensity I_0 and alternating bright and dark fringes of lower intensities (intensities of the first left and the first right equal $0.05I_0$). To analyze the diffraction pattern, it is convenient to divide the slit into two halves, as shown in Figure 3.3 a. Keeping in mind that all the

waves are in phase as they leave the slit, consider rays 1 and 2. As these two rays travel toward a viewing screen far to the right of the figure, ray 1 travels farther than ray 2 by an amount equal to the path difference $(a/2) \sin \phi$ where a is the width of the slit.

Similarly, the path difference between rays 2 and 3 is also $(a/2) \sin \phi$. If this path difference is exactly half a wavelength (corresponding to a phase difference of 180°), then the two waves cancel each other and destructive interference is obtained.

This is true for any two rays that originate at points separated by half the slit width because the phase difference between two such points is 180° . Therefore, waves from the upper half of the

slit interfere destructively with waves from the lower half when $\sin \phi = \pm \frac{\lambda}{a}$.

If we divide the slit into four, six, eight ... equal parts and use similar reasoning, we find that corresponding point on the viewing screen is dark. Therefore, the general condition for

destructive interference is $\sin \phi = \pm \frac{k\lambda}{a}$ $k=0,1,2,3, \dots$. The general features of the intensity distribution are shown in Figure 3.3 b.

3.3 Diffraction grating and its formula

The diffraction grating, a useful device for analyzing light sources, consists of a large number of equally spaced parallel slits. A *transmission grating* can be made by cutting parallel lines on a glass plate with a precision ruling machine. The spaces between the lines are transparent to the light and hence act as separate slits. A *reflection grating* can be made by cutting parallel lines on the surface of a reflective material. The reflection of light from the spaces between the lines is specular, and the reflection from the lines cut into the material is diffuse. Thus, the spaces between the lines act as parallel sources of reflected light, like the slits in a transmission grating. Gratings that have many lines very close to each other can have very small slit spacings. For example, a grating ruled with 5000 lines/cm has a slit spacing $d = 2 \cdot 10^{-4}$ cm.

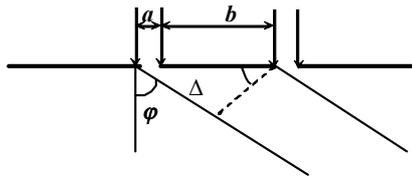


Fig. 3.4

3.4 note that the path difference Δ between rays from any two adjacent slits is equal to $\Delta = (a + b)\sin\phi = d \sin\phi$. If this path Δ difference equals one wavelength or some integral multiple of a wavelength, then all waves from all slits are in phase at point of observation and a bright fringe is observed there. Therefore, the condition for maxima in the interference pattern at the angle ϕ is $d \sin\phi = k\lambda$ where $k=0, 1, 2, 3, \dots$ is the order of maximum.

We can use this expression to calculate the wavelength if we know the grating spacing and the angle at which the maximum of particular order k is observed.

In 1913, Max von Laue (1879–1960) suggested that the regular array of atoms in a crystal could act as a three-dimensional diffraction grating for X-rays. Subsequent experiments confirmed this prediction. The diffraction patterns are complex because of the three-dimensional nature of the crystal. Nevertheless, X-ray diffraction has proved to be an invaluable technique for elucidating crystalline structures and for understanding the structure of matter. The condition for maxima in the reflected X-ray beam is known as **Bragg's law**, after W.L. Bragg (1890–1971), who first derived formula which can be used to calculate the spacing between atomic planes, if the wavelength and diffraction angle are measured. This relationship is :

$$2d \sin\theta = m \lambda, \quad m = 1, 2, 3, \dots$$

here d is distance between two parallel crystalline planes, θ is angle between crystalline plane, created by atoms and X-rays.

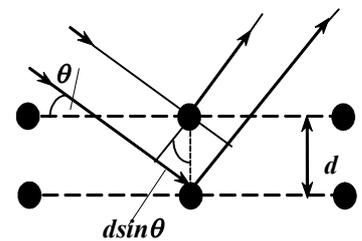


Fig.3.3

4. Polarization and dispersion of light waves

4.1. Polarisation

Above we described the transverse nature of light and all other electromagnetic waves. Polarization is firm evidence of this transverse nature.

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a wave having some particular orientation of the electric field vector \vec{E} , corresponding to the direction of atomic vibration.

The *direction of polarization* of each individual wave is defined to be the direction in which the electric field is vibrating.

An individual electromagnetic wave could have its \vec{E} vector in the y - z plane, making any possible angle with the y axis. Because all directions of vibration from a wave source are possible, the resultant electromagnetic wave is a superposition of waves vibrating in many different directions. The result is an unpolarized light beam, represented in Fig. 4.1. The direction of wave propagation in this figure is perpendicular to the page. The arrows show a few possible directions of the electric field vectors for the individual waves making up the resultant beam. At any given point and at some instant of time, all these individual electric field vectors add to give one resultant electric field vector.

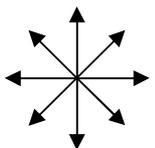


Fig.4.1

A wave is said to be linearly polarized if the resultant electric field \vec{E} vibrates in the same direction at all times at a particular point, as shown in Figure 4.2. Sometimes, such a wave is described as *plane-polarized*. The plane formed by \vec{E} and the direction of propagation is shown on Fig. 4.2. If the wave in Figure 4.2 represented the resultant of all individual waves, the plane of polarization is the x - y plane.

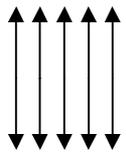


Fig.4.2

It is possible to obtain a linearly polarized beam from an unpolarized beam by removing all waves from the beam except those whose electric field vectors oscillate in a single plane.

The polarized light from unpolarized light in the following processes.

1. Polarization by selective absorption in special material, called *polaroid*,

Polaroid transmits waves whose electric fields vibrate in a plane parallel to a certain direction and absorbs waves whose electric fields vibrate in all other directions.

2. Polarization by double refraction.

Using materials (calcite, quartz), which are characterized by different light velocity in different directions and, hence, two indices of refraction (Fig.4.3).

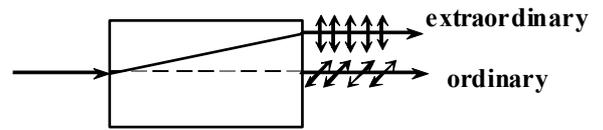


Fig. 4.3

3. Polarization by reflection.

It is found that the parallel component of electric field \vec{E} reflects more strongly than the perpendicular component, and this results in a partially polarized reflected beam. Furthermore, the refracted beam is also partially polarized.

4. Laser sources of polarized light.

Laser are source of stimulated radiation, which is coherent and polarized due special electron structure of laser's material.

4.2 Malus's law

When polarized light of intensity I_0 is emitted by a polarizer and then incident on an analyzer, the light transmitted through the analyzer has an intensity equal to $I_0 \cos^2 \varphi$, where φ is the angle between the polarizer and analyzer transmission axes (fig.4.4).

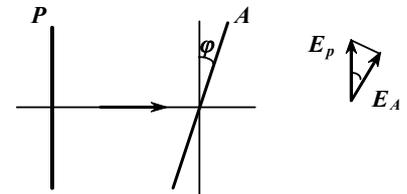


Fig.4.4

If an incident light is unpolarized, then adding transmitted components of all the direction of incident oscillations gives exactly one half of the incident light intensity.

4.3 Brewster's law

In general, reflected light is partially polarized. However, reflected light is completely polarized when the angle of incidence is such that the angle between the reflected and refracted beams is 90° (fig. 4.5). This angle of incidence, called the polarizing angle i , satisfies Brewster's law:

$$\text{tg } i = n,$$

where n is the refraction index of the reflecting medium.

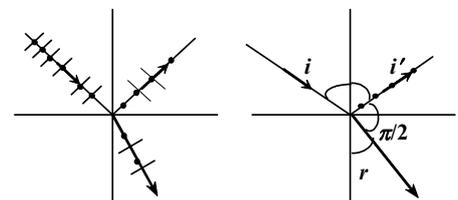


Fig.4.5

4.4. Dispersion. Spectrum of dispersion

The dependence of the index of refraction on wavelength is called **dispersion**. Because n is a function of wavelength, Snell's law indicates that **the angle of refraction made when light enters a material depends on the wavelength of the light**.

The index of refraction for a material usually decreases with increasing wavelength. This means that violet light ($\lambda=400$ nm) refracts more than red light ($\lambda =650$ nm) when passing from air into a material.

Because of dispersion, the different colours refract through different angles of deviation, and the rays that emerge from the second face of the prism (fig 4.4) spread out in a series of colors known as a visible **spectrum**.

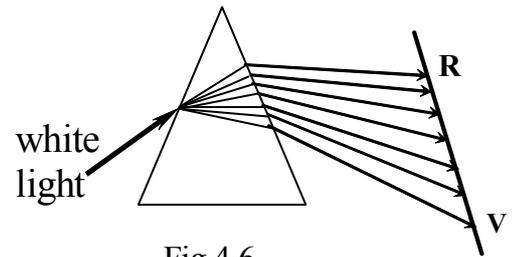


Fig.4.6

Prisms are often used in an instrument known as a **prism spectrometer**. All hot, low-pressure gases emit their own characteristic spectra. Thus, one use of a prism spectrometer is to identify gases. For example, sodium emits only two wavelengths in the visible spectrum: two closely spaced yellow lines. Likewise, mercury vapour has its own characteristic spectrum, consisting of four prominent wavelengths – orange, green, blue, and violet lines – along with some wavelengths of lower intensity. The particular wavelengths emitted by a gas serve as “fingerprints” of that gas. Spectra of substances depend on type and concentrations of atoms. So the spectrum of liquids is more complex than spectrum of gases, and have a view of strips. Spectrum of solids is most complex. It is **continuous**. The simplest atom’s spectrum is the spectrum of hydrogen.

Spectral analysis, which is the measurement of the wavelengths emitted or absorbed by a substance, is a powerful general tool in many scientific areas.

To explain the dependence of the index of refraction on frequency (or wavelength) one may to use the electron theory of substance and electromagnetic theory of light.

Consider that electric field vector projections for electromagnetic (light) waves

$$E = E_m \cos(\omega t + \alpha).$$

When the electromagnetic wave is passing through the substance, the force $F = -eE = -eE_m \cos(\omega t + \alpha)$ acts on every electron of the substance. As result, electron starts to oscillate.

The equation of electron’s motion in such case is:

$$m\ddot{x} = -kx + eE_m \cos(\omega t + \alpha),$$

$$\ddot{x} = -\frac{kx}{m} + \frac{eE_m}{m} \cos(\omega t + \alpha),$$

here x is displacement of electron from the balance position.

After solution of this differential equation the following formula for amplitude of electron’s oscillations is obtained:

$x_0 = \frac{eE_m}{m(\omega_0^2 - \omega^2)}$. Here x_0 is amplitude of electron’s oscillations, ω_0 is natural cycle frequency of electron, ω is the cycle frequency of external force

$eE = eE_m \cos(\omega t + \alpha)$.

From electrostatics we know that dipole moment of molecules depends on it’s arm. In our case it is the electron’s displacement x . As $x = x(\omega)$, dipole moment

$P = \frac{\sum q_i x_i}{\Delta V}$ depends on frequency too. But dipole moment is connected with relative permittivity ϵ (electric permeability), which, in its turn, is connected with refraction index: $n \approx \sqrt{\epsilon}$ (see chapter 1.2).

As result, the chain of dependences $P(\omega) \rightarrow \epsilon(\omega) \rightarrow n(\omega)$ is created, which shows that refraction index is a function of

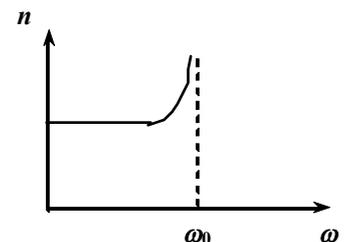


Figure 4.5

frequency. For the normal dispersion the experimentally observed $n(\omega)$ dependence it has the form shown in fig.4.5

Problem 1.

Absolute refraction index of the diamond $n = 2.4$ and of the glass $n = 1.5$. Calculate the ratio between the width of substrates from these materials, if the time of light propagation in both is the same ?

Strategy :

Use the relation between absolute refraction index and speed of light $n = \frac{c}{g}$. Assume that incident angle is zero.

Solution

1) Time of light propagation in substrate $t = \frac{x}{g}$; From problem situation it is known that: $t_1 = t_2$,

so

2) $\frac{x_1}{g_1} = \frac{x_2}{g_2}$; and then

3) $\frac{x_d}{x_g} = \frac{g_2}{g_1}$;

4) $n = \frac{c}{g}$;

After substitution one receives the required ratio :

5) $\frac{x_d}{x_g} = \frac{c \cdot n_g}{n_d \cdot c} = \frac{n_g}{n_d} = \frac{1.5}{2.4} = \frac{5}{8}$.

The required ratio is five to eight.

Problem 2.

Find a slit spacing of diffraction grating, if the third maximum is situated on distance $x = 4$ cm from center of image on screen .The distance between screen and diffraction grating $l = 1.5$ m (figure 4.6)

Strategy. Use formula for diffraction grating Take into account geometry's factor of problem situation: ratio between distances x and l . (figure 4.6).

Solution

As formula for maximum of diffraction grating is $d \sin \varphi = n \lambda$ and angle φ is small one receives:

$$d = \frac{n \lambda}{\sin \varphi} \approx \frac{n \lambda l}{x} = \frac{3 \cdot 0.589 \cdot 10^{-6} \cdot 1.6}{0.04} \approx 0.7 \cdot 10^{-4} \text{ m}$$

Problem 3.

Newton's rings appears as a series of light and dark rings formed due to interference between the light waves reflected from the top and bottom surfaces of the air film (as a rule), formed between the convex lens of large curvature and adjacent glass plate.

Determine absolute refraction index of liquid, if in Newton's ring arrangement the space between lens and glass plate is filled by the liquid. The observation take place in incident light

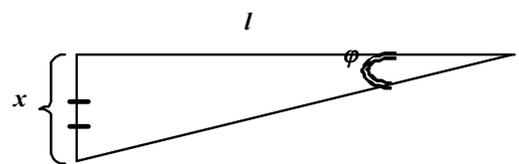


Figure 4.6

with wavelength $\lambda=589$ nm. The radius of curvature of the convex lens $R=10$ m and the radius of the third light ring $r=3.65$ mm.

Strategy.

Read the theory for laboratory experiment O2 Determination of light wavelength using Newton's rings. Find condition for maximum in incident light and radius of the Newton's ring.

Solution.

1) If the space between lens and glass plate is filled with the liquid then in the condition of maximum in incident light one has to consider the liquid refraction coefficient n . So for the layer width h between the lens and glass plate one can write:

a) from one side : $2 h n = m \lambda$;

b) from another side (geometry of Newton's ring picture): $h = r_m^2 / 2R$.

2) Then $n r_m^2 / R = m \lambda$ and $n = m \lambda R / r_m^2$

3) After substitution numeric data , absolute refraction coefficient is calculated. $n=1.3$

Problem 4

Calculate the angle between polarizer and analyzer (Figure 4.4), if intensity of unpolarized light I_0 after passage through polarizer and analyzer is one quarter of initial value.

Strategy.

Malus's law application in the case, when an incident light is unpolarized.

Solution

1) After passage through polarizer intensity of unpolarized light $I_1=0.5 I_0$.

2) After passage through analyzer intensity of light $I_2=0.5 I_0 \cos^2 \varphi$

3) From problem condition $I_2/ I_0=0.25$, so :

4) $\cos^2 \varphi = 0.25/0.5=1/2$ and the angle between polarizer and analyzer $\varphi= 45^\circ$

5. Blackbody radiation and Plank's hypothesis

5.1. Thermal radiation. Wien's and Stefan's laws

Fundamental constituents of matter are electrical charges, in particular, electrons. Motion of these electrons, according to Maxwell's theory, is accompanied by electromagnetic radiation. There are various types of radiation emitted by real bodies, among them thermal radiation and different kinds of luminescence (chemi-luminescence, electro-luminescence, photo-luminescence), X ray emission, radioactive γ decay, etc. An object at any temperature emits electromagnetic radiation at expense of its internal energy. This type of radiation is called **thermal radiation**. Thermal radiation plays unique role as it is the only equilibrium type of radiation. This means that a body can reach equilibrium with its surrounding by emitting and absorbing thermal radiation.

All objects radiate energy continuously in the form of electromagnetic waves due to thermal vibrations of their molecules. These vibrations (for example) create the orange glow of an electric stove burner, an electric space heater, and the coils of a toaster.

To characterize the thermal radiation, an integral characteristic R_T , called radiance, is introduced. The radiance R_T equals power emitted from the unit surface of a hot body. In SI, unit radiance is W/m^2 .

The rate at which an object radiates energy is proportional to the fourth power of its absolute temperature. This is known as Stefan's law, expressed in equation

$$R_T = e \sigma T^4$$

where σ is the Stefan-Boltzmann constant, equal to $5.6696 \cdot 10^{-8} W/m^2 \cdot K^4$, e is a constant called the emissivity of the object, and T is the object's absolute temperature (in Kelvins). The value of e can vary between zero and one, depending on the properties of the object's surface.

When an object is in equilibrium with its surroundings, it radiates and absorbs energy at the same rate, so its temperature remains constant. When an object is hotter than its surroundings, it radiates more energy than it absorbs and so cools. As an object radiates energy at a rate given by as Stefan's law, it also absorbs radiation. If it didn't, the object would eventually radiate all its energy and its temperature would reach absolute zero. The energy an object absorbs comes from its environment, which consists of other bodies that radiate energy. If an object is at a temperature T , and its surroundings are at a temperature T_0 , the net energy gained or lost each second by the object with the surface area S as a result of radiation is:

$$P = e \sigma S (T^4 - T_0^4).$$

Kirchhoff discovered that for any body the ratio of the emissivity to absorption rate $\frac{e}{a} = \varphi(\lambda, T)$

remains constant, dependent only on of radiation and temperature of the body.

An **ideal absorber** is an object that absorbs all the light radiation incident on it, including invisible infrared, ultraviolet radiation and other components of electromagnetic spectrum. Such an object is called a **black body** because a room-temperature black body would look black. Since a black body doesn't reflect radiation at any wavelength, any light emitted by it is due to atomic and molecular vibrations alone. An opening in the cavity shown in fig. 5.1 is a good model of a black body. As light enters the cavity through the small opening, part is reflected and part is absorbed on each reflection from the interior walls. After many reflections, essentially all of the incident energy is absorbed.

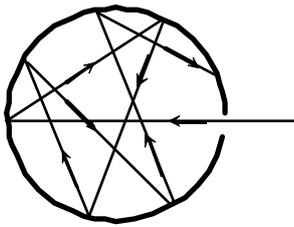


Fig.5.1

An ideal absorber is also an ideal radiator of energy. A perfect black body has emissivity $e = 1$. The Sun, for example, is nearly a perfect black body. This statement may seem contradictory, because the Sun is bright, not dark; however, the light that comes from the Sun is emitted, not reflected. Black bodies are perfect absorbers that look black at room temperature because they don't reflect any light. All black bodies, except those at absolute zero, emit light that has a characteristic spectrum, to be discussed later. In contrast to black

bodies, an object for which $e = 0$ absorbs none of the energy incident on it, reflecting it all. Such a body is an **ideal reflector**.

The spectrum of the radiation depends on the temperature and properties of the object. At low temperatures, the wavelengths of the thermal radiation are mainly in the infrared region and hence not observable by the eye. As the temperature of an object increases, the object eventually begins to glow red. At sufficiently high temperatures, it appears to be white, as in the glow of the hot tungsten filament of a lightbulb.

Energy is not evenly distributed in the electromagnetic spectrum. At any temperature there is a wavelength at which maximum energy emission is observed. This wavelength determines the visible colour of the glowing body. Experimental data for the distribution of energy in blackbody radiation at five different temperatures are shown in Figure 5.2. The radiated energy varies with wavelength and temperature. As the temperature of the blackbody increases, the total amount of energy it emits (area under the curve) increases. Also, with increasing temperature, the peak of the distribution shifts to shorter wavelength. This shift obeys the following relationship, called **Wien's displacement law**, $\lambda_{max} = \frac{b}{T}$ where λ_{max} is the wavelength at which the curve peaks and T is the absolute temperature of the object emitting the radiation. Constant $b = 2.9 \cdot 10^{-3} \text{ m} \cdot \text{K}$.

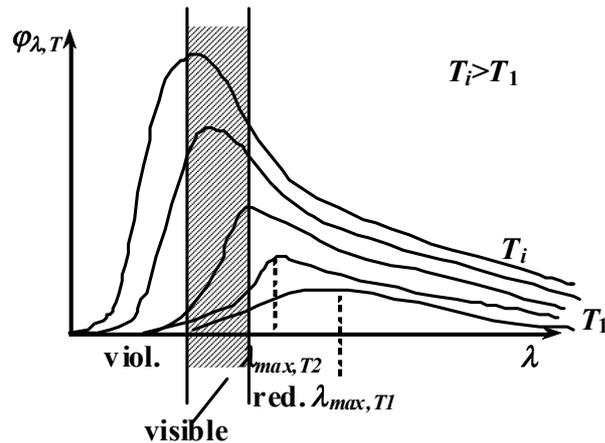


Fig. 5.2

For example, approximately 1340 J of electromagnetic radiation from the Sun passes through each square meter at the top of the Earth's atmosphere every second. This radiation is primarily visible light, accompanied by significant amounts of infrared and ultraviolet. Some of this energy is reflected back into space, and some is absorbed by the atmosphere, but enough arrives at the surface of the Earth.

The amount of energy radiated by an object can be measured with temperature-sensitive recording equipment via a technique called thermography. For example, thermography makes it possible to see one's environment with or without visible illumination using the radiation emitted by objects with near room temperature.

Non-contacting temperature determination of an object's surface may be realized by the devices, called pyrometers. There are the brightness (disappearing filament) pyrometers, using Stefan's law and the color (ratio) pyrometers. Color pyrometers based on comparing of heat illumination in different spectrum. A few examples of pyrometers applications are:

1. Thermo energetic-measurements and temperature control of sections inaccessible for other types of measuring devices;
2. Electroenergetic control and fire safety;
3. Construction – determination of heat losses in pipeline, walls of building ,etc.

5.2. Plank's hypothesis

By the end of the 19th century, it had become apparent that the classical theory of thermal radiation was inadequate. The basic problem was in understanding the observed distribution energy as a function of wavelength in the radiation emitted by a black body. Attempts to use classical ideas to explain the shapes of the curves shown in Figure 5.2 failed. At long wavelengths, classical theory is in good agreement with the experimental data. At short wavelengths, however, major disagreement exists between classical theory and experiment. As wavelength λ approaches zero, classical theory predicts that the amount of energy being radiated should increase. In fact, the theory erroneously predicts that the intensity should be infinite, when the experimental data shows it should approach zero. This result is called the **ultraviolet catastrophe**, because theory and experiment disagree strongly in the short-wavelength, ultraviolet region of the spectrum.

In 1900 Planck proposed a formula for blackbody radiation that was in complete agreement with experiments at all wavelengths, leading to a curve shown in Figure 5.2. Planck hypothesized that blackbody radiation was produced by submicroscopic charged oscillators, which he called *resonators*. He assumed that the walls of a glowing cavity were composed of billions of these resonators, although their exact nature was unknown. The resonators were allowed to have only certain discrete energies

$$\varepsilon = n\hbar\omega$$

where n is a positive integer called a quantum number, ω is the circle frequency of vibration of the resonator, $\hbar = \frac{h}{2\pi}$ and $h=6,63 \cdot 10^{-34}$ J·s is a constant known as **Planck's constant**.

Because the energy of each resonator can have only discrete values given by Planck's formula we say that the energy is **quantized**. Each discrete energy value represents a different **quantum state**, with each value of n representing a specific quantum state. When the resonator is in the $n = 1$ quantum state, its energy is $\hbar\omega$; when it is in the $n = 2$ quantum state, its energy is $2\hbar\omega$ and so on.

The key point in Planck's theory is the assumption of quantized energy states. This is a radical departure from classical physics, the "quantum leap" that led to a totally new understanding of nature. A theory based on the quantum concept (rather than on classical concepts) had to be used to explain a number of other phenomena at the atomic level.

Using the idea of quantum resonators Planck also derived a formula for intensity of radiation (emissivity) as function of circular frequency:

$$f(\omega, T) = \frac{\hbar\omega^3}{4\pi^2c^2} \cdot \frac{1}{\exp(\hbar\omega/kT) - 1}$$

Here $f(\omega, T)$ is intensity of radiation, ω is circular frequency, k is Boltzman's constant, T is temperature in Kelvins, c is velocity of light in vacuum. As the function of wavelength λ this expression for intensity has the form:

$$\varphi(\lambda, T) = \frac{4\pi^2\hbar \cdot c^2}{\lambda^5} \cdot \frac{1}{\exp(2\pi\hbar c/\lambda \cdot kT) - 1}$$

and perfectly reproduces experimental dependences shown in fig. 5.2.

5.3 Photoeffect. Laws of photoeffect

5.3.1 External photoelectrical effect.

In the latter part of the 19th century, experiments showed that light incident on certain material surfaces caused the emission of electrons. This phenomenon is known as **the external photoelectric effect**, and the emitted electrons are called **photoelectrons**. The first discovery of this phenomenon was made by Hertz. Besides external photoeffect, there is **internal photoeffect**, when light incident on certain surfaces caused increasing of material conductivity due to increasing of free carrier's concentration. The variant of internal photoeffect is photovoltaic effect, when due to special technology p-n junction is created in material (this will be explained in Sect. 10.3). Across this junction potential difference appears at illumination.

In a figure 5.3 a schematic diagram of a photoelectric effect apparatus is shown. An evacuated glass tube known as a photocell contains a metal electron emitter Em connected to the negative terminal of a variable power supply. Another metal plate, electrode Col (the collector), is maintained at a positive potential by the power supply. When emitter Em is illuminated by light having a wavelength shorter than some particular wavelength that depends on the material used to make emitter Em, a current is detected

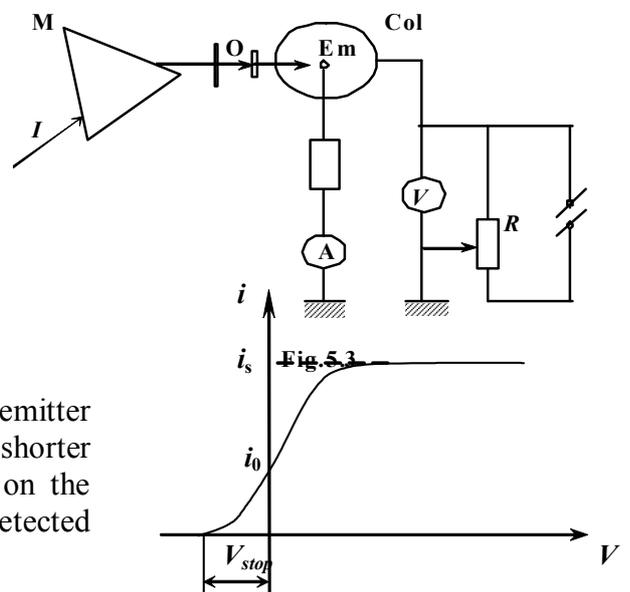


Fig.5.4

by the ammeter, indicating a flow of charges across the gap between electrodes Em and Col. This current arises from photoelectrons emitted from the negative electrode Em and collected at the positive electrode Col. Figure 5.4 is a plot of the photoelectric current versus the potential difference V between Em and Col for two light intensities. At large values of V , the current reaches a maximum value. In addition, the current increases as the incident light intensity increases. Finally, when V is negative – that is, when the power supply in the circuit is reversed to make Em positive and Col negative – the current drops to a low value because most of the emitted photoelectrons are repelled by the now negative electrode Col. In this situation, only those electrons having a kinetic energy greater than the magnitude of $e U$ reach the electrode Col, where e is the charge of the electron.

When V is equal to or more negative than V_{stop} , called the **stopping potential**, no electrons reach Col and the current is zero. The stopping potential is *independent* of the radiation intensity. The maximum kinetic energy of the photoelectrons is $e V_{\text{stop}}$.

5.3.2 The laws of photoelectric effect.

The laws of the photoelectric effect can't be explained within classical physics or the wave theory of light.

1. The current increases with light intensity, reaches a saturation level for large values of U and current of saturation is proportional to light flow: $i_{\text{sat}} = \gamma \Phi$
2. No electrons are emitted if the incident light frequency is below some **threshold frequency** $f_t = W_{\text{fun}}/h$; where W_{fun} - work function of electron, which represents the minimum energy with which an electron is bound in the metal, h - Planck's constant. Sometimes instead the threshold frequency threshold wavelength is used. It is maximum wavelength of incident light, when photoelectrical effect take place.
3. The maximum kinetic energy of the photoelectrons is independent of light intensity. The maximum kinetic energy of the photoelectrons increases with increasing light frequency. According to Einstein, the maximum kinetic energy for these liberated photoelectrons may be determined from formula: $\hbar\omega = W_{\text{fun}} + \frac{m g_m^2}{2}$; where $\hbar\omega$ is photon's energy, W_{fun} is work

function, $\frac{m g_m^2}{2}$ is the maximum kinetic energy of the photoelectrons.

Einstein extended Planck's concept of quantization to electromagnetic waves and explained the previously mentioned features of the photoelectric effect that cannot be understood on the base of classical physics. He suggested that a tiny packet of light energy or **photon** would be emitted when a quantized oscillator made a jump from an energy state $E_n = nh\nu$ to the next lower state $E_{n-1} = (n - 1)h\nu$. Conservation of energy would require the decrease in oscillator energy, $h\nu$, to be equal to the photon's energy E . The key point here is that the light energy lost by the emitter, $h\nu$, stays sharply localized in a tiny packet or particle called a photon. In Einstein's model, a photon is so localized that it can give *all* its energy $h\nu$ to a single electron in the material. Photoelectrons are created by absorption of a single photon, so the energy of that photon must be greater than or equal to the work function, else no photoelectrons will be produced. This explains the threshold frequency.

From equation $\hbar\omega = W_{\text{fun}} + \frac{m g_m^2}{2}$ one obtains that the maximum kinetic energy of the photoelectrons depends only on the frequency of the light and the value of the work function. This explains photoelectron's energy independence of light intensity. Photons are specific particles which have the energy, linear momentum, spin equal to 1, but have no electrical charge and rest mass.

The photoelectric effect has many interesting applications. Among others, it is used in a device called the **photocell** transforming the energy of light into electricity. There are vacuum

photocells and solid state photocells. Photocells may serve as source of energy or as photoresistors, which change their resistance under the light action.

Photomultipliers also use the photoelectric effect. In these light-sensitive vacuum tubes a photocathode is coated materials with low work function, in particular caesium, rubidium and antimony. Even if the photocathode is illuminated by almost invisible amount of light, it releases electrons. These electrons are accelerated by electric field inside the vacuum tube and are gradually increased in number in series of secondary emissions from additional electrodes (dynodes). In such a way very weak light signals are amplified and detected. Now photomultipliers are widely used in night vision devices, detectors of ionizing radiation and numerous medical equipment designs (blood analysis by determining the relative concentration of various components in blood, etc).

Photoelectric image sensors, such as charge-coupled devices (CCD), are widely used today in digital color imaging (for example, in photo and video cameras). CCD is a semiconductor device and an internal photoeffect is realised in it (to be discussed in chapter devoted to properties of semiconductors).

Photoelectron spectroscopy is a powerful tool which enables scientists to characterize new materials. Since the energy of the photoelectrons emitted is exactly the energy of the incident photon minus the material's work function or binding energy, the work function of a sample can be determined by bombarding it with X-rays or ultra-violet light and measuring the kinetic energy distribution of the electrons emitted.

Problem 1

Find the work function W_{fun} and threshold frequency f_i of vacuum phototube, if under irradiation by the light with the frequency $1.6 \cdot 10^{15}$ Hz the current disappeared at the stopping voltage $V = 4.1$ Volts.

Strategy.

Analyze phenomena of external photoeffect and use Einstein's formula of photoeffect.

$$\hbar\omega = W_{\text{fun}} + \frac{m g_m^2}{2}$$

Solution

When voltage V is equal to negative stopping voltage V_{stop} , no electrons reach Collector of vacuum photo tube and the current is zero. That is mean : 1) $\frac{m g_{\text{max}}^2}{2} = eV_{\text{stop}}$.

2) After substitution the value of electron charge and stopping voltage to step 1) the value of kinetic energy $\frac{m g_{\text{max}}^2}{2} = 4.1$ eV is found.

3) The next calculation: $\hbar\omega = h\nu = 6.63 \cdot 10^{-34} \cdot 1.6 \cdot 10^{15} \frac{1}{1.6 \cdot 10^{-19}} = 6.63$ eV;

4) $W_{\text{fun}} = h\nu - eV = (6.63 - 4.1) = 2.53$ eV

Problem 2.

How many photons are emitted by the filament of light bulb, if the power of light bulb $P = 1$ W and average wavelength of radiation $\lambda = 1$ micrometer?

Strategy.

Apply of photon theory

Solution.

1) Power of light bulb radiation equals energy of photon E multiplied by number of photon n emitted in 1 second: $P = E \cdot n$

$$2) E = \hbar \omega = \hbar \frac{2\pi c}{\lambda}$$

$$3) n = \frac{P\lambda}{2\pi\hbar c}$$

After substitution of all quantities expressed in SI units one receives $n \approx 5 \cdot 10^{18}$ photons/s.

5.4 X-Rays

X-rays are produced when high-speed electrons are suddenly slowed down—for example, when a metal target is struck by electrons that have been accelerated due application of a potential difference of several thousand volts between electrodes (Fig.5.5).

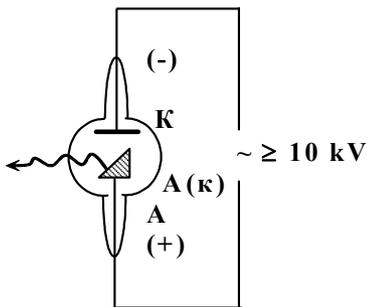


Fig.5.5

X-ray tube is vacuum tube with filament (cathode) and metallic target (anode, called anticathode). A current in the filament causes electrons to be emitted, and these freed electrons are accelerated toward a dense metal target, such as tungsten, which is held at a higher potential than the filament.

Typical X-ray wavelengths are about 0.1 nm, which is of the order of the atomic spacing in a solid. Due to high frequencies, X-rays easily penetrate most materials.

The spectrum of radiation emitted by an X-ray tube has two distinct components. They have different nature and take place at different vacuum tube voltage.

5.4.1 Braking radiation

1. One component (fig.5.6) is a **continuous broad spectrum** that depends on the voltage applied to the tube. The continuous radiation is called in German **bremstrahlung**, a word meaning “braking radiation”, because electrons emit radiation when they undergo an acceleration inside the target. The explanation of the continuous part of X-ray spectrum is the following.

As the electron passes close to a positively charged nucleus contained in the target material; hence, it undergoes an acceleration. From classical physics we know that any charged particle will emit electromagnetic radiation when it is accelerated. (An example of this phenomenon is the production of electromagnetic waves by accelerated charges in a radio antenna, as described early.) According to quantum theory, this radiation must appear in the form of photons. Because the radiated photon carries energy, the electron must lose kinetic energy because of its encounter with the target nucleus. An extreme example would consist of the electron losing all of its energy in a single collision. In this case, the initial energy of the electron the energy (eU) of the electron after it has been accelerated through a potential difference of U volt is transformed completely into the energy of the photon which has maximum frequency.

$$\text{In equation form } eU = \hbar\omega_{\max} \text{ or } eU = \frac{\hbar \cdot 2\pi c}{\lambda_{\min}}$$

From this formula the shortest wavelength radiation that can be produced, λ_{\min} is:

$$\lambda_{\min} = \frac{2\pi\hbar c}{e} \cdot \frac{1}{U}$$

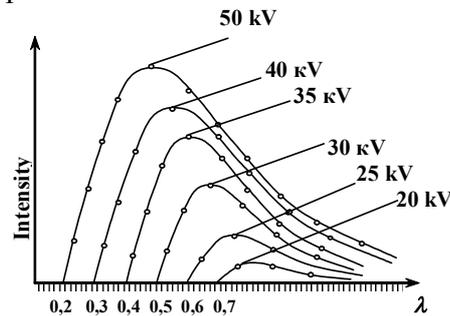


Fig.5.6

5.4.2 Characteristic radiation

The second components of X-rays is called **characteristic radiation**. Superimposed on this component is a series of sharp, intense lines that depend on the nature of the target material (Fig.5.7).

The accelerating voltage must exceed a certain value, called the **threshold voltage**, in order to observe these sharp lines, which represent radiation emitted by the target atoms as their electrons undergo rearrangements.

These discrete lines, called **characteristic X-rays**, were discovered in 1908, but their origin remained unexplained until the detailed theory of atomic structure were developed. The wavelength of **characteristic X-rays** depends on atomic number of material and number of shell in atom. Short explanation of characteristic radiation appearance is the following.

Electron, emitted by cathode, collides with an electron in an inner shell of a target atom with sufficient energy to remove the electron from the atom. The vacancy created in the shell is filled when an electron in a higher level drops down into the lower energy level containing the vacancy.

The transition is accompanied by the emission of a photon with energy equal to the difference between the two levels. Typically, the energy of such transitions is greater than 1 000 eV, and the emitted X-ray photons have wavelengths in the range of 0.01 nm to 1 nm. The frequency ν of characteristic radiation is determined by Mozley's law :

$$\sqrt{\nu} = b(Z - \sigma),$$

where Z is atomic number of material, σ is shield constant, dependent on number of inner shells, b is constant dependent on the design of X-ray tube.

Another formula of Mozley's law, in quantum form: $\omega = (z - \sigma)^2 R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$.

Here R is Rydberg constant. n_1, n_2 are quantum numbers of electron levels, accomplished during emission of characteristic X-rays.

X-ray can be used in medicine, biology, material science, construction engineering to determine their quality, etc. The method of crystalline structure determination by X-ray diffraction was thoroughly developed by W. H. Bragg and his son W. L. Bragg. Since then, thousands of crystalline structures have been investigated. Most importantly, the technique of X-ray diffraction has been used to determine the atomic arrangement of complex organic molecules such as proteins. Proteins are large molecules containing thousands of atoms that help to regulate and speed up chemical life processes in cells. The main technique used to determine the molecular structure of proteins, DNA, and RNA is X-ray diffraction using X-rays of wavelength of about 1.0 Å. This technique allows to "see" atoms that are separated by about this distance in molecules.

5.5 The Compton effect

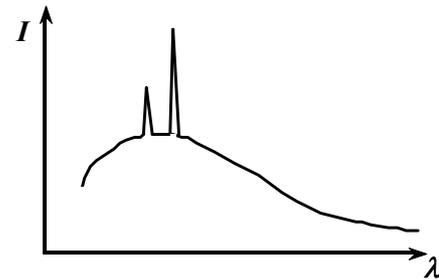


Fig.5.7

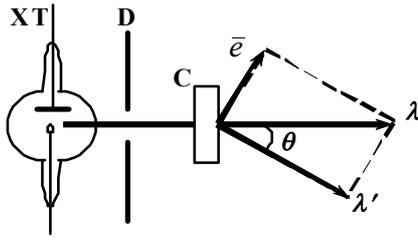


Fig.5.5

Further justification for the photon nature of light came from an experiment conducted by professor Arthur H. Compton in 1923. In his experiment, Compton directed an X-ray beam of wavelength λ toward a block of graphite (Fig.5.5). He found that the scattered X-rays had a slightly longer wavelength λ_1 than the incident X-rays, and hence the energies of the scattered rays were lower. The amount of energy reduction depended on the angle at which the X-rays were scattered. The change $\Delta\lambda$ in wavelength between a scattered X-ray

and an incident X-ray is called the **Compton shift**.

In order to explain this effect, Compton assumed that if a photon behaves like a particle, its collision with other particles is similar to a collision between two billiard balls. Hence, the X-ray photon carries both measurable *energy* and *momentum*, and these two quantities must be conserved in a collision. If the incident photon collides with an electron initially at rest, the photon transfers some of its energy and momentum to the electron. As a result, the energy and frequency of the scattered photon are lowered and its wavelength increases. We can express this processes in form of laws of conservation :

$$m_0c^2 + \hbar\omega = E_e + \hbar\omega' = c\sqrt{p_e^2 + m_0^2c^2} + \hbar\omega' \quad (\text{conservation of energy})$$

$$\hbar\vec{K} = \hbar\vec{K}' + \vec{P}_e \quad P_f = \frac{\hbar\omega}{c} = \hbar K \quad (\text{conservation of linear momentum})$$

momentum)

Applying relativistic energy and momentum conservation to the collision described, the shift in wavelength of the scattered photon as $\Delta\lambda = \lambda_c(1 - \cos\theta)$

Here Compton wavelength

$$\lambda_c = \frac{2\pi\hbar}{mc}$$

here m is mass of electron, c is light velocity in vacuum, θ is angle between the directions of the scattered and incident photons. Experimental results for X-rays scattered from various targets obey equation for Compton shift $\Delta\lambda$, characteristic X-rays and photoelectrical effect strongly support the photon concept.

Experiments, described in this chapter, are fundamental proofs of particle theory of light.