



## *MECHANICS AND MATERIALS SCIENCE*

## *МЕХАНІКА ТА МАТЕРІАЛОЗНАВСТВО*

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### **PROBABILISTIC MODELING OF PHYSICAL DAMAGE PROCESSES OF FIBER-REINFORCED COMPOSITE PLATES UNDER DYNAMIC LOADING**

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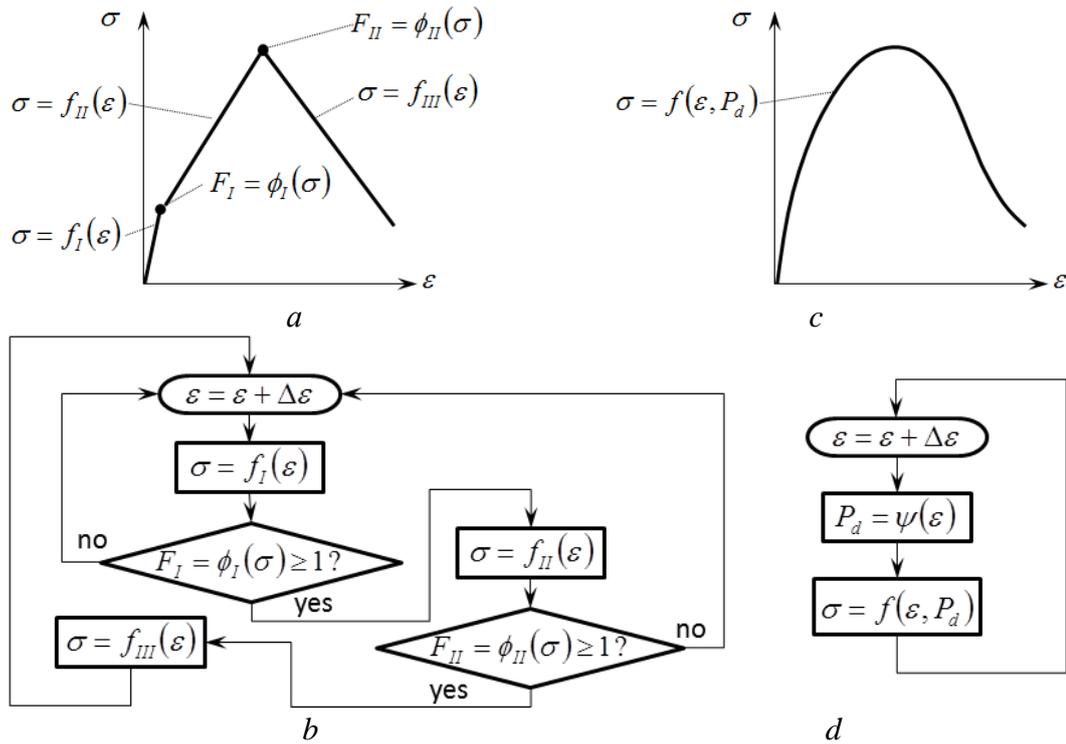
*Summary.* Probabilistic material model for strength analysis of a composite taking into account the evolution of damage of bonds between its structural elements based on the analysis of physical damage processes of the internal structure of composites and considered earlier phenomenological model of impact in collision velocity range from 20 to 1500 m/s was proposed. The comparison of results obtained by other authors was done against experimental data for the case of tension of a fiber-reinforced material at strain rates from 0,441 to 44,1 s<sup>-1</sup>.

**Key words:** fiber-reinforced composites, damage evolution, probabilistic modeling, high-velocity impact, strain rate.

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**Problem setting.** Traditional approach to composite materials modeling under static and dynamic loading is based on developed earlier for metals and other generally isotropic constructive materials method of analysis of mathematically deterministic relationship between arising stresses and strains  $\sigma = f(\epsilon)$  in constructive element. The given dependence is different in different ranges of stresses/strains taking into account temperature and interaction rate or ignoring their change. To differentiate separate stages of deforming, which are in fact caused by the prevailing influence on strength of different mechanisms of damage of internal structure of analyzed material, criteria of strength  $F_i$ , which are functions of current stress  $F_i = \phi_i(\sigma_i)$  and explicitly divide characteristic areas of strain diagram between each other, are introduced (Fig. 1, a).

Such criteria discern area of deforming and area of irretrievable material fracture in the simplest case. In another cases, additional criteria of initiation of each investigated type of fracture  $F_I$ ,  $F_{II}$  etc. are introduced, that gives opportunity to predict the behavior of material more precisely but requires preliminary defining of a greater number of model parameters. While performing computation, it is taken into account that during a separate considered time segment under the given force action and under conditions of the given kinematic constraints some strain  $\Delta\epsilon$  is implemented, which should be added to initial strain  $\epsilon_0$ , already available at the moment of the beginning of the current time segment. It gives the possibility to find stress  $\sigma = f_i(\epsilon) = f_i(\epsilon_0 + \Delta\epsilon)$  and check if the condition of the strength criterion  $F_i$  is implemented, after that, if it is necessary, the value of stress is recalculated according to the dependence  $\sigma = f_{i+1}(\epsilon)$ , which is correspondent to a new area of the common curve.



**Figure 1.** General algorithm of analysis of construction material behavior under loading: deterministic (a, b) and probabilistic (c, d) approaches:  $\sigma$  – stress;  $\varepsilon$  – strain;  $\Delta\varepsilon$  – strain increment;  $f_i(\varepsilon)$  – piecewise continuous dependencies of stresses on strains defined in ranges as  $f_I(\varepsilon)$ ,  $f_{II}(\varepsilon)$ ,  $f_{III}(\varepsilon)$ ;  $F_i$  – strength criteria that define the boundaries of function ranges  $f_i(\varepsilon)$ ;  $\phi_i(\sigma)$  – strength criterion dependence on the current stress;  $P_d$  – probability of bonds fracture;  $\psi(\varepsilon)$  – dependence of the probability of bonds fracture on current strain

The problem arising at modeling of constructive materials behavior under loading is caused by its multi-component structure and diversity of active fracture mechanisms, complexity of unification and taking into account of all factors influencing deformation and fracture processes and combining them to uniform equations represented within model of material in the form of dependences of stress on strain.

Stress, as a force characteristic, being a measure, which is used for mathematical description of basic for macro-world influence of electromagnetic and gravitational interactions, is definitely connected in the model with strain as with a space measure of change of mutual arrangement of points of material.

The aim of this investigation is an attempt of a uniform description of the given relation with the main physical both space and time characteristics of motion and interaction taking into account complex inner structure of composite material.

As it will be described below, it is thought that this task can be solved using so called probabilistic approach. In literature probabilistic approach is often understood as calculation, which takes into account the initial statistical spread of parameters, which is present due to technological peculiarities of material production, such as rectilinearity of fibers fitting, density of local areas of material and thus local spread of strength parameters, etc. In this investigation, probabilistic approach is understood as a model, which takes into account that strength of a material depends on availability of chemical and physical bonds between atoms and molecules of material, which are destructed with some probability  $P_d$  and are recovered between the same particles or already another ones; besides, this probability is defined by surrounding conditions

and influence of environment, including the influence of an interacting with the material of accelerated striker under impact loading. The algorithm depicted in Fig. 1, b illustrates the idea of using such general approach. As stated above, the probability  $P_d$  of fracture of bonds between the material structural elements, which define the strength of this material (further for short – probability of bonds fracture) depends on conditions of environment, current state of material, including its strain, that is  $P_d = \psi(\varepsilon)$ , and rate of strain and temperature.

**Probabilistic modeling of damage processes.** Earlier, on a small sample of data the attempt of research data modeling in through-out breaking of hybrid fiber composites using probabilistic strength criteria  $F_{PSC}$  was done in the following way [1]:

$$F_{PSC}(v_0, x_i, t) = K_p \left( \frac{W_{\Sigma}^{EME}(v_0) - W_m^{EME}(v_0)}{W_{\Sigma}^{AE}(v_0)} \right) \cdot P_{PSC}(x_i, t) = \begin{cases} 0 \\ 1 \end{cases}, \quad (1)$$

where  $v_0$  – impact rate,  $W_{\Sigma}^{EME}$  and  $W_{\Sigma}^{AE}$  – energy of electromagnetic and acoustic emission for the whole range of investigated frequencies,  $W_m^{EME}$  – energy of electromagnetic emission at selected multiple frequencies,  $P_{PSC}$  – parameter depending on coordinate  $x_i$  characterizing distance to the point of initial contact of a specimen with an impactor and time of interaction  $t$ ,  $K_p$  – parameters of modeling using Monte Carlo method. Correspondent phenomenological model was obtained and checked for the case of impact within impact velocity range of from 20 to 1500 m/s, which is why further considerations are applicable primarily for the referred conditions of impact interaction.

General procedure of using strength criteria is as follows: if some combination of values, which characterize material deformation and damage and which are in this case the emission parameters, is observed, then material is fractured (the criteria is equal to unity). If the combination does not occur (the criteria is equal to zero) – the material can still resist loading and undergo further deforming and damage increasing. To take into consideration the processes of fracture and recovery of bonds in material, which define the emission, it is necessary to investigate peculiarities of damage formation in details.

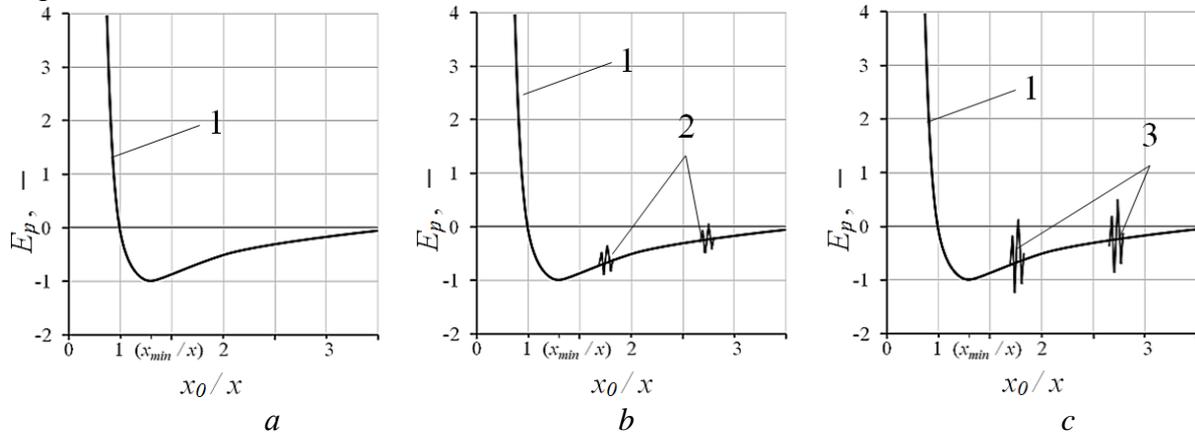
Damage of material represents a breach of bonds between its structural elements, moreover recovering or forming new bonds is also possible, but it probably takes place at already modified mutual location of the bonded elements. Such bonds are caused inter alia by the balance of Van der Waals forces of interaction of the induced dipole moments and forces of interatomic repulsion according to Pauli exclusion principle [2]. In case of interaction of structural elements (atoms and molecules of fiber and matrix) such relation can be described by different potentials. If to neglect the variety of covalent bonds in material, influence of molecular polarity, configuration of atomic surrounding etc., then within the given task essential peculiarities of particles' behavior depending on the distance between them can be described by Lennard-Jones potential [3], which represents a model of a pairwise molecular interaction describing dependence of interaction energy between two particles on the distance between them.

Correspondent interaction energy of particles of mass  $m$  may be represented according to this model in the following way [4]:

$$E_p = 4 \cdot \xi \cdot m \cdot \left[ \left( \frac{x_0}{x} \right)^{12} - \left( \frac{x_0}{x} \right)^6 \right] = \xi \cdot m \cdot \left[ \left( \frac{x_{\min}}{x} \right)^{12} - 2 \cdot \left( \frac{x_{\min}}{x} \right)^6 \right], \quad (2)$$

where  $x$  – distance between centers of particles,  $\xi$  – depth of potential well,  $x_0$  – distance, at

which interaction energy becomes equal to zero. Parameters  $\xi$  and  $x_0$  are characteristics of atoms of corresponding substance. A minimum of potential energy is in point  $x_{\min} = \sqrt[6]{2} \cdot x_0$  (Fig. 2).



**Figure 2.** Dependence of the interaction energy of two material particles on the relative distance between them in statics (*a*) and in case of presence of small (*b*) or significant (*c*) energy fluctuations of one of the interacting particles: 1 – curve according to (2); 2 – small deviations of interaction energy, for example, due to raise of the temperature; 3 – significant deviations of interaction energy, for example, due to an impact

Stockmayer, Buckingham, Morse potentials and in more complicated cases Tersoff and Brenner potentials can be used [4]. Stockmayer potential is worth of investigation because of the presence of dipole moments in the interface layers of composite materials or because of their forming in the process of consolidation of hybrid fiber prepreps by pressing at elevated temperatures [5]:

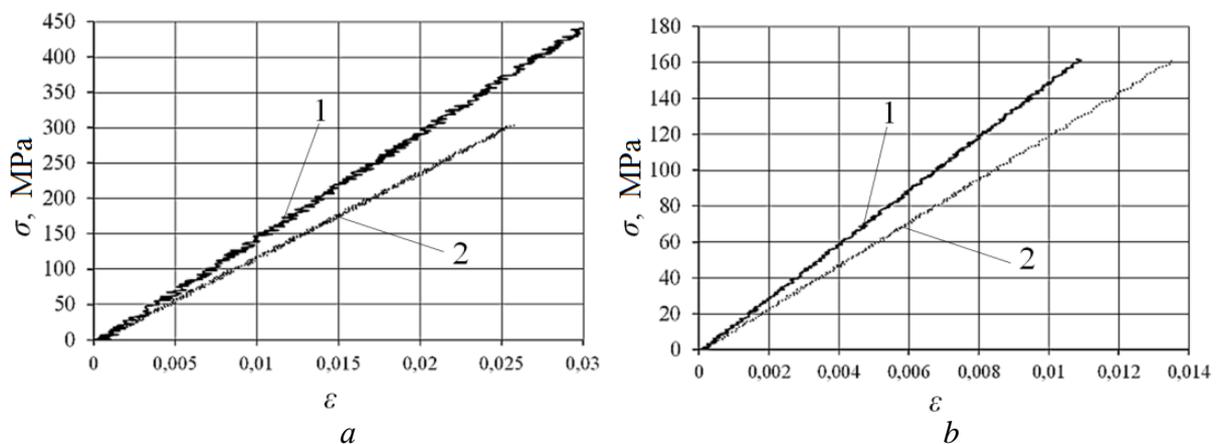
$$E_{nSt} = E_p - m \cdot \frac{m_a \cdot m_b}{x^3} \cdot (2 \cdot \cos \delta_a \cdot \cos \delta_b - \sin \delta_a \cdot \sin \delta_b \cdot \cos(\gamma_a - \gamma_b)) \quad (3)$$

where  $E_p$  – Lennard-Jones potential, for  $i = a, b$   $m_i$ ,  $\delta_i$  and  $\gamma_i$  – dipole moments, polar angles and azimuth angles of molecules  $a$  and  $b$  of mass  $m$ . Its use is reasonable for composites made of polymers with essentially non-polar covalent bonds, which are not compensated by symmetry of links of the polymer chain, for example, for polyvinylchloride or cellulose and starch matrices used for bio-degradable plastics. In polypropylene, dipole moments of groups C–H and C–CH<sub>3</sub> are practically equal, that is why links of the polymer chain  $[-CH_2-CHCH_3-]_n$  have a very small dipole moment equal to 0.364 Debye or  $1.21 \cdot 10^{-30}$  C·m. In polystyrene, dipole moment of each link is also insignificant and approximately equal to 0.301 Debye [6]. Thus, the influence of the second term in (3) can be neglected in these cases for the great bulk of matrix material but not in the interface layer of matrix-fiber contact. Use of potentials according to (2), (3) or choice of equation of another type matters for specification of peculiarities of crystal or amorphous structure of certain substances, but the damage simulation is mostly influenced by the considered below general form of the dependence of interaction energy on the distance between particles.

Potential energy increases when structural elements are moving away from each other. On its certain level, the interacting force can be considered insignificant. Then neglecting the force, the bond can be considered to be split, or fractured. Thus the value of maximum strain can be obtained, at which the bond is considered to be still not fractured. In practice in molecular dynamics calculations, the value  $x_c = 2.5 \cdot x_0$  is often accepted as the appropriate strain level [7]:

$$\frac{E_p}{m \cdot \xi} \approx 1.63\% . \quad (4)$$

However, when calculating complex heterogeneous materials, which have multiscale structure (e.g. for metals – atomic scale, scale of grains etc., for composites – scale of atoms, molecules, molecular clusters, taking into account influence of the interface etc.), derivation of the value of critical strain from detailed model of inner structure taking into account characteristic energetic parameters of interactions at each scale and their interrelation appears to be a rather complicated process. That is why it is necessary to base the model on experimental data characterizing integrally the result of interaction of damages at different scales, which are obtained according to standard methods dependences  $\sigma_i = f(\varepsilon_j)$  [8]. For example, for textile reinforced and two-directionally reinforced composites on the basis of propylene, the mentioned characteristics were obtained by authors using standard industrial equipment according to standards ISO 527-5 and ISO 14126 [9]. Mechanic diagrams for tension and compression of tested materials are shown in Figure 3.



**Figure 3.** Dependence of stresses  $\sigma$  on strains  $\varepsilon$  for tests of material specimens of both investigated types in accordance to ISO 527-5 under tension (*a*) and compression (*b*): 1 – textile-reinforced material; 2 – two-directionally reinforced material

For many fiber-reinforced materials, the values of strains at fracture ranges from 2 to 2.5%. Similarly to data in Fig. 3 this value appears to be really close to the result obtained in (4).

Not only the level of potential energy of the mutual arrangement of material particles (i.e. their spatial interaction), but also their kinetic energy (i.e. temporal interaction of particles) influences the fracture of bonds between them [10, 11]. The influence of possible fluctuations of the kinetic energy, which take place due to change of the temperature, on overcoming the energy of bond fracture at different strain levels is illustrated in Figure 2, *b* in comparison to the influence of fluctuations owing to introduction of additional kinetic energy into the system during impact in analogical conditions, which is illustrated in Figure 2, *c*. As it is shown in the Figures, small fluctuations can result in reaching the zero value of interaction energy, i.e. to the bond fracture, only if particles are remote enough from each other, that is only in the case of considerable initial strains. Bigger values of fluctuations can cause bond breaking even in the case of small initial strains. The reasons of this effect are investigated below in more details.

Even insignificant fluctuation of the element energy (as it is shown in Figure 2, *b*) can cause real bond fracture at big strains. However, at small strains it is necessary to achieve more significant level of kinetic energy for bond breaking, which can be reached e.g. at impact. For this case, the kinetic energy should correspond to the energy of the element leaving the potential well:

$$E_k = E_n(x_{\min}) = \xi \cdot m, \quad (5)$$

where  $m$  – mass of the element. In case of interaction of material with an impactor, the given energy is a result of influence of both the impulse, which is introduced during the impact, and of the kinetic energy of particles, which is initially available in the system and corresponds to the material temperature.

In such case, the level of energy, which is sufficient for nonreversible damage of bonds, corresponds to a critical temperature of the material  $T_c$ . Depending on the type of fractured bonds, this temperature may be equal to the identically called critical temperature, which is defined as a temperature at which substance preserves its gaseous state at arbitrary pressure without transition into liquid, that is formation of normal bonds between substance particles is practically impossible, and correspondingly the probability of fracture of such bonds (let us call it  $P_f$ ) exceeds the probability of their recovery (let us call it  $P_r$ ):

$$P_f > P_r, \quad (6)$$

or according to an alternative approach, the probability of bond formation is actually equal to zero:

$$\begin{cases} P_f \rightarrow 1, \\ P_r \rightarrow 0. \end{cases} \quad (7)$$

Let us also understand the normal bonds as bonds between atoms or molecules of substance causing normal stresses under loading. Analogically, tangential bonds are understood as bonds, which cause generation of tangential stresses under application of loading. As a classical liquid, unlike a solid body, does not resist tangential stresses, temperature of melting  $T_m$  can be considered as the critical temperature  $T_c$ , at which probability of fracture of tangential bonds exceeds the probability of their recovery. Transition from solid state of matter into liquid one can be considered as damage of initial arranged solid body structure of the material. Thus liquid can be considered as a damaged solid material with fractured tangential bonds:

$$\begin{cases} (P_f^\tau - P_r^\tau) \geq 0, \\ (P_f^\sigma - P_r^\sigma) \leq 0, \end{cases} \quad (8)$$

where index  $\tau$  indicates the probability of fracture  $P_f$  and recovery  $P_r$  of tangential bonds, index  $\sigma$  – of normal bonds in such meaning as it was stated above. As it is known, the boiling temperature of liquid depends on pressure, which in turn is related to the strain  $\varepsilon$ . Here the influence of both factors is observed, which are the temperature  $T$  and strain  $\varepsilon$ . Moreover, for effecting an actual bond breakage, the probability of fracture should be implemented both according to the first factor  $P_f(T)$  and to the second one  $P_f(\varepsilon)$ . Correspondingly the resulting probability of  $P_f(T, \varepsilon)$  should be equal to the product of probabilities of each influencing factor. The same holds for probability of bond recovery  $P_r(T, \varepsilon)$ :

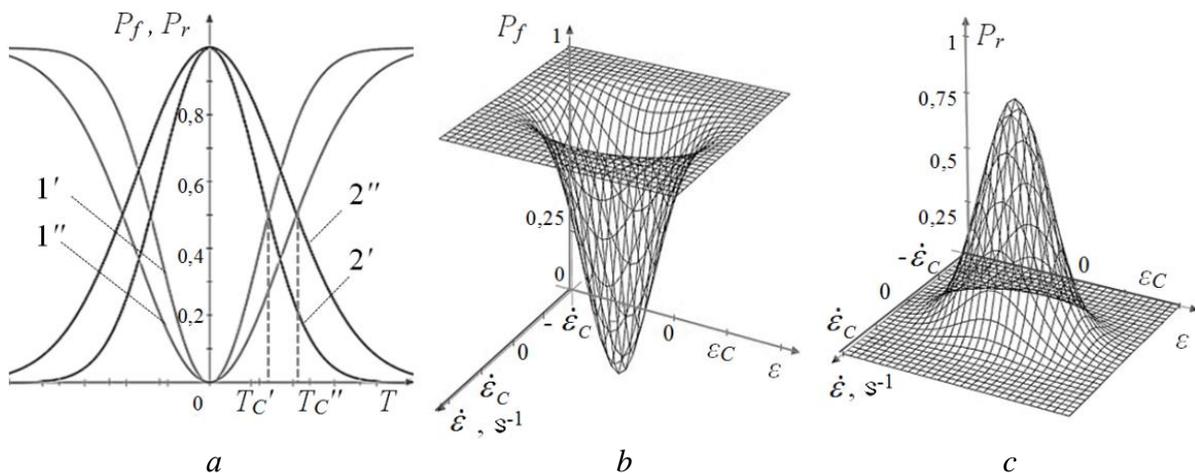
$$\begin{cases} P_f(T, \varepsilon) = P_f(T) \cdot P_f(\varepsilon), \\ P_r(T, \varepsilon) = P_r(T) \cdot P_r(\varepsilon). \end{cases} \quad (9)$$

The influence of pressure on the melting temperature should be also taken into account, that is why correction of reference physical data for substances should be made before simulation.

The described above approach can be illustrated by theoretical curves of probability of bond fracture  $P_f$  and recovery  $P_r$ , which are shown in Fig. 4 and received in the following way. It is suggested that any temperature-related fluctuations of kinetic energy of molecules or atoms are absent at temperature  $T = 0$  K, hence the probability of fracture of bonds between them (in the absence of strains and external loading) is equal to zero, it corresponds to the curves (1) in Figure 4, *a*.

With increasing temperature kinetic energy of material molecular or atomic motion increases, but not only the very value of this energy defines if the bond is fractured, but also many random factors such as mutual arrangement of particles, the influence of neighboring particles etc. Thus it is suggested that the specific value of this probability is described by a Gaussian function, which is normalized in order to conform with the known evidence, that at critical temperature  $T_c$  the probability of fracture of bonds exceeds the probability of their recovery (or probability of forming a new bond) and in a small time period (taking the time into consideration is obligatory and will be examined in more details in future investigations) almost all bonds appear to be damaged.

Hence, graphs in Fig. 4 show not the probability density function but mathematical expectancy of the probability  $P_f$  and  $P_r$ . Thus dependence of the probability of bond recovery  $P_r$  on temperature gives a value of unity at  $T = 0$  K, does not exceed the value of  $P_f$  at  $T < T_c$  and is described by a scaled Gaussian function, similarly to the dependence of  $P_f(T)$ . Left branches of the curves (1) and (2) in Fig. 4, *a* are shown for clarity, though only right branches have physical sense at  $T > 0$  K.



**Figure 4.** Dependence of the probability of fracture  $P_f$  (1) and recovery  $P_r$  (2) of bonds between structural material elements at various critical temperatures  $T_c$  on the absolute temperature  $T$  (*a*), strain  $\varepsilon$  and strain rate  $\dot{\varepsilon}$  (*b*, *c*)

Consequently, the difference of the curves of probability  $P_f$  of bond fracture, which are shown in Figure 4, *a* and each of which corresponds to bonds of a certain type, is explained with the issue, that each type of bonds is characterized with a different critical temperature  $T_c$ .

Analogically to dependences  $P_f$  and  $P_r$  on temperature, theoretical dependences  $P_f$  and  $P_r$  on strain  $\varepsilon$  and strain rate  $\dot{\varepsilon}$  were constructed using appropriate critical values  $\varepsilon_c$  and  $\dot{\varepsilon}_c$  (for compression and tension). In Figure 4,  $b$ ,  $c$  corresponding surfaces are shown. Such approach allows explaining many peculiarities of material behavior, which are described in [12 – 17].

Owing to mentioned multiscale structure of composite materials, taking into account the aggregate influence of destroyed bonds at micro-, meso- and macroscales and corresponding characteristic energies of their fracture is rather complicated [18]. But a definite velocity of mutual motion of particles is correspondent to each value of fracture energy [19, 20]. Such the critical value is in this case the speed of sound  $v_c$ , which is the velocity, exceeding of which causes formation of discontinuity surface and probability of recovery of bonds between structural elements becomes actually equal to zero. If it refers to damage of tangential bonds (as they were described above), then the speed  $v_t$  of transverse sound waves propagation defines the material behavior. If it refers to damage of normal bonds, then the speed  $v_l$  of longitudinal sound waves propagation is decisive. Appropriate strain values can be calculated (but a problem arises of additional investigation of the influence of size of a reference tested specimen for obtaining correct final data) or linear velocity values can be used straightforwardly without recalculation.

If normality of the probability  $P_f$  distribution is assumed, as it was done above, then it can be described by the Gauss function of the general form [21]:

$$P_f(\varepsilon, \dot{\varepsilon}, T) = \frac{1}{2\pi \cdot L} \cdot e^{-\left( \frac{(\varepsilon - d\mu_\varepsilon)^2}{2 \cdot (dS_\varepsilon)^2} + \frac{(\dot{\varepsilon} - d\mu_{\dot{\varepsilon}})^2}{2 \cdot (dS_{\dot{\varepsilon}})^2} + \frac{(T - d\mu_T)^2}{2 \cdot (dS_T)^2} \right)}, \quad (10)$$

where  $L = dS_\varepsilon \cdot dS_{\dot{\varepsilon}} \cdot dS_T$ ,  $d\mu_i$  – shift of the probability function peak relative to the axis of strain  $\varepsilon$ , strain rate  $\dot{\varepsilon}$ , temperature  $T$  for  $i = \varepsilon, \dot{\varepsilon}, T$  correspondingly,  $dS_i$  – root-mean-square fluctuation for indicated arguments, which can be represented as a ratio of appropriate material characteristics, which describe the critical state of some type of bonds, to some coefficient  $n_i$ , which characterizes the given critical state:

$$dS_i = \frac{i_c}{n_i}, \quad i = \varepsilon, \dot{\varepsilon}, T, \quad (11)$$

where for each  $i$  there is  $\varepsilon_c$  – critical value of strain, which corresponds to referred above  $x_c$  as to an indicator of potential strain energy;  $\dot{\varepsilon}_c$  is critical value of strain rate as of an indicator of directed (unidirectional) kinetic energy, which is introduced into the system of material particles;  $T_c$  is critical value of temperature as of an indicator of kinetic energy, which is diffused in the material (and related to different spatial directions). Thus, for example, if to assume that 99.9% of particles are in gaseous state at critical temperature, then coefficient  $n_i$  is equal to three (according to the so-called rule of three sigma), if only 64.2% are assumed to be in gaseous state, then it is equal to unity. Formula (10) can be modified in case of taking into account more than three investigated factors of influence on the probability of fracture. And on the contrary, if the influence of e.g. temperature is neglected, the equation for the graph of function, which is shown in Figure 4,  $b$ , will be as follows:

$$P_f(\varepsilon, \dot{\varepsilon}) = \frac{1}{2\pi \cdot L_{\varepsilon, \dot{\varepsilon}}} \cdot e^{-\left(\frac{\varepsilon^2}{2 \cdot (dS_\varepsilon)^2} + \frac{\dot{\varepsilon}^2}{2 \cdot (dS_{\dot{\varepsilon}})^2}\right)}, \quad (12)$$

where  $L_{\varepsilon, \dot{\varepsilon}} = dS_\varepsilon \cdot dS_{\dot{\varepsilon}}$ , another symbols are correspondent to the previous ones.

Taking into account suggestion about the possibility of usage of Gauss function for description of the probability dependence on investigated parameters  $\varepsilon, \dot{\varepsilon}, T$ , the probability function  $P_r$  of recovery of material bonds can be represented analogically to (10) in the following way:

$$P_r(\varepsilon, \dot{\varepsilon}, T) = 1 - \frac{1}{2\pi \cdot L} \cdot e^{-\left(\frac{(\varepsilon - d\mu_\varepsilon)^2}{2 \cdot (dS_\varepsilon)^2} + \frac{(\dot{\varepsilon} - d\mu_{\dot{\varepsilon}})^2}{2 \cdot (dS_{\dot{\varepsilon}})^2} + \frac{(T - d\mu_T)^2}{2 \cdot (dS_T)^2}\right)}, \quad (13)$$

where symbols of values are correspondent to ones stated above and the given equation takes into account, that zero probability of bond fracture corresponds to unity probability, that all bonds in material are recovered. In (10) and (13) the following values are assumed:

$$d\mu_\varepsilon = d\mu_{\dot{\varepsilon}} = d\mu_T = 0. \quad (14)$$

The surface describing the dependence of probability of bond recovery  $P_r(\varepsilon, \dot{\varepsilon})$  on strain  $\varepsilon$  and strain rate  $\dot{\varepsilon}$  without taking into consideration the temperature, is shown in Figure 4, *c* and it is built using the next dependence:

$$P_r(\varepsilon, \dot{\varepsilon}) = 1 - \frac{1}{2\pi \cdot L_{\varepsilon, \dot{\varepsilon}}} \cdot e^{-\left(\frac{\varepsilon^2}{2 \cdot (dS_\varepsilon)^2} + \frac{\dot{\varepsilon}^2}{2 \cdot (dS_{\dot{\varepsilon}})^2}\right)}, \quad (15)$$

which was derived from (13) taking into account (14) and assuming

$$\begin{cases} T \equiv 0, \\ dS_T \equiv 1. \end{cases} \quad (16)$$

The resulting probability function, let us call it  $P_d$ , which defines the probability of damage as the probability of material bonds being fractured and then non-recovered, can be represented by the next formula:

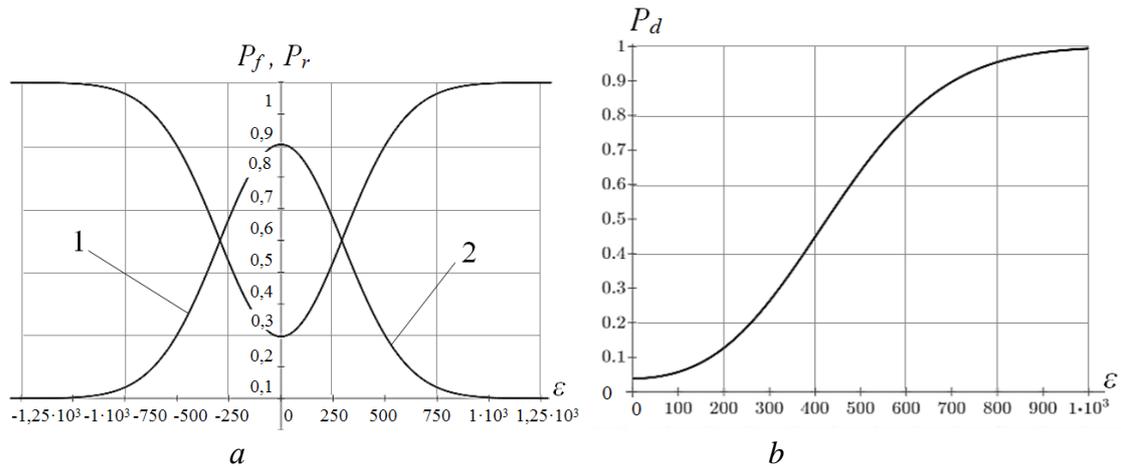
$$P_d(\varepsilon, \dot{\varepsilon}, T) = (1 - P_r(\varepsilon, \dot{\varepsilon}, T)) \cdot P_f(\varepsilon, \dot{\varepsilon}, T), \quad (17)$$

or at given temperature and velocity it can be represented in the following way:

$$P_d|_{\dot{\varepsilon}, T} = (1 - P_r(\varepsilon)|_{\dot{\varepsilon}, T}) \cdot P_f(\varepsilon)|_{\dot{\varepsilon}, T}, \quad (18)$$

besides, the current strain rate  $\dot{\varepsilon}$  and temperature  $T$  are considered as parameters, or variables.

Intersection of surfaces describing the probability functions of bond fracture and recovery, which is correspondent to temperature of experiment execution  $T = 20^\circ\text{C}$ , is shown in Figure 5, *a*. For this case the dependence of non-recovery of bonds, that is of probability of forming final damage, is shown in Figure 5, *b*.



**Figure 5.** Dependences of the probability of recovery  $P_r$  and fracture  $P_f$  of bonds (a), as well as the probability of formation of the final damage  $P_d$  (b), corresponding to the temperature  $T = 20^\circ\text{C}$  during the experiment execution, on the strain  $\varepsilon$

Thus, the authors suggest to use the proposed probability functions  $P_f$ ,  $P_r$ ,  $P_d$  within the model framework as it was described above and to calculate damage  $d$ , which determines the components of tensor  $D_{ij}$  for calculating stresses using formula (1), as a specific implementation of the probability value (17) or (18), that is damage  $d$  can be defined in the following way:

$$d|_{\dot{\varepsilon}, T} = (1 - r(\varepsilon)|_{\dot{\varepsilon}, T}) \cdot f(\varepsilon)|_{\dot{\varepsilon}, T}, \quad (19)$$

where  $r$  and  $f$  – specific implementations of probability values equal to the part of recovered and damaged material bonds with probabilities  $P_r$  and  $P_f$  correspondingly.

From Figure 5, a it is obvious that at a non-zero temperature even in case of absence of strains or introduced kinetic energy, the damage probability is not equal to zero, though it is close to it. It explains material creep under even small static loading: during very small periods of time, the material, which is damaged to the extent of  $f$ , undergoes deforming. But then its bonds, and thus also its strength, is actually recovered, what is depicted by the model with the value of  $r$ . In such case the total final damage is equal to  $d$ .

It should be taking into account, that strains and strain rates, just like their referent, i.e. corresponding to initial non-damaged material, values, in different directions of an anisotropic composite can be different at the same moment of time  $t$ . They result thus in a set of values  $d_{ij}$ , each for a different direction, instead of one value  $d$ . Tensor of material damage parameters for an orthotropic layer of a laminated plastic according to [22-24] is as follows:

$$D_{ij} = \begin{pmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & D_{12} \end{pmatrix} = \begin{pmatrix} 1/(1-d_{11}) & 0 & 0 \\ 0 & 1/(1-d_{22}) & 0 \\ 0 & 0 & 1/(1-d_{12}) \end{pmatrix}, \quad (20)$$

where  $D_{ij}$  – damage tensor elements,  $i, j = 1, 2$ ,  $d_{ij}$  – material damage in the direction of warp fibers ( $d_{11}$ ), weft fibers ( $d_{22}$ ) of a textile (bidirectional) reinforced composite, and in the plane

of the laminated plastic ( $d_{12}$ ).

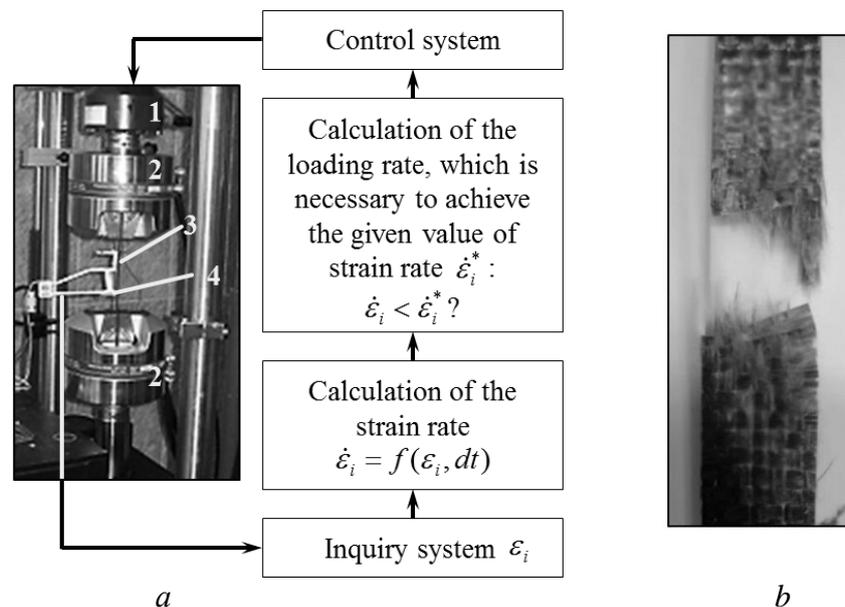
Tensor of non-damaged initial stiffness  $C_{ij}$  of the orthotropic laminated composite in expanded form [25]:

$$C_{ij} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}, \quad i, j = 1, 2, \quad (21)$$

where values of components  $C_{11}$ ,  $C_{12}$ ,  $C_{22}$ ,  $C_{21}$  are found according to known mechanical properties of the composite, namely the Young modulus in the warp fiber  $E_1$  and weft fiber  $E_2$  direction, Poisson's ratios  $\nu_{12}$ ,  $\nu_{21}$ ;  $G_{12}$  is shear modulus. Represented in (1) vectors of stresses and strains can be explicitly written in the following way [23, 25]:

$$\sigma_i = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}, \quad \varepsilon_j = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}. \quad (22)$$

**Software implementation of the mathematical model and comparison to experiment.** Testing of the suggested model has been done for the case of standard experiments according to [8] (Fig. 6).



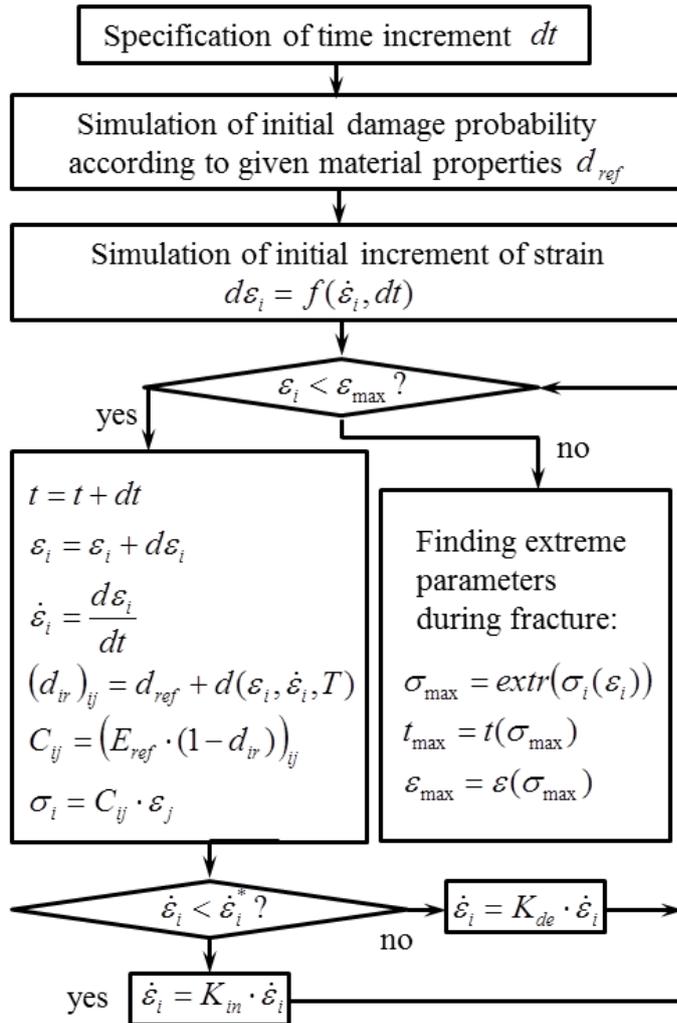
**Figure 6.** Scheme of the experiment at constant strain rate (a) and general view of the specimen destroyed during testing (b); 1 – specimen capture drive, 2 – specimen grips, 3 – specimen, 4 – strain sensor

To achieve this aim, an algorithm has been programmatically implemented, which allowed simulation of an experimental test providing the strain rate  $\dot{\varepsilon}$  being constantly equal to some given value. In a real experiment such the type of testing is typically implemented using extensometers and a high frequency data acquisition system, which receives data from sensors and immediately regulates loading rate in order to provide almost constant strain rate value.

Requirements to the algorithm included support of non-immediate accelerating of the

loading system, wherever possible a smooth support of constant strain rate as a reaction to material model behavior, finishing of the simulation after reaching of given parameters of strain or loading. That is, the conditions of a real experiment execution were taken into account to achieve a correct comparative evaluation of results.

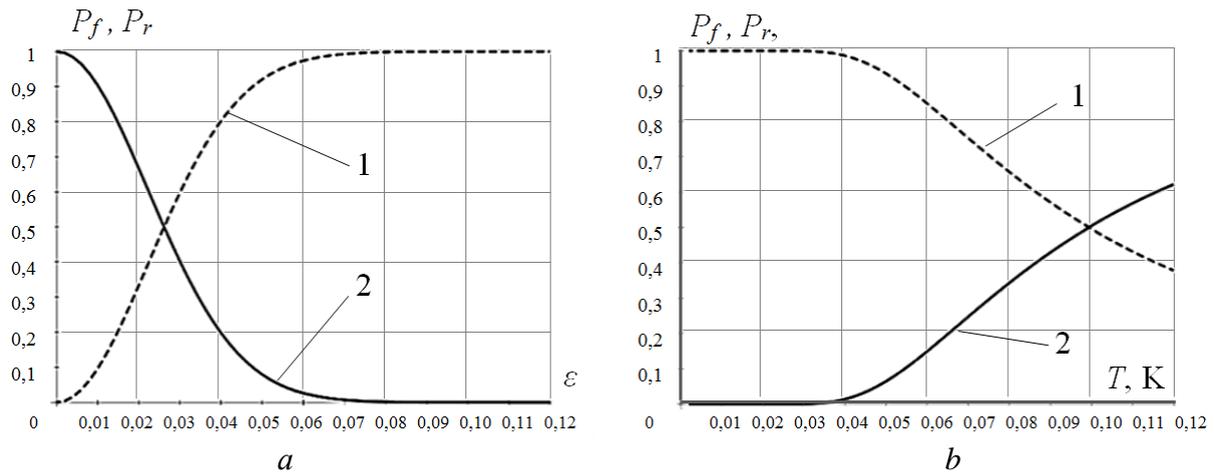
Software algorithm of model implementation is shown in Fig. 7. Determination of unknown output parameters, such as critical strain and critical temperature of a given tested material, was implemented with the help of mathematical optimization methods on the base of known physical parameters of the given material and according to suggested model hypothesis. This optimization was an inseparable part of the very modeling algorithm.



**Figure 7.** Algorithm of software implementation of material model under loading at a constant strain rate  $\dot{\varepsilon}^*$  :  
 $d$  – current material damage under applied loading;  
 $t$  – time;  
 $\varepsilon$  – strain;  
 $\varepsilon_{\max}$  – strain, at which the specimen fracture occurs;  
 $\dot{\varepsilon}$  – strain rate;  
 $\dot{\varepsilon}^*$  – given in experiment strain rate;  
 $T$  – temperature;  
 $C$  – material stiffness;  
 $E_{ref}$  – reference value of the modulus of elasticity;  
 $\sigma$  – stresses in the material;  
 $K_{in}$  – coefficient indicating what increment the corrected value of strain will have in the next time step (positive increment or increase in the case of  $K_{in}$  and negative increment or decrease in the case of  $K_{de}$ )

Strain  $\varepsilon_{\max}$  figures as a parameter of material at first and represents the strain at which material fracture occurs. Correction of this value is done according to testing results after preliminary simulations. New value of strain  $\varepsilon_{\max}$  is accepted to be equal to the strain, at which during the test an extremum of the stress-strain curve  $\sigma_{\max}(\varepsilon)$  is observed. Besides,  $t_{\max}$  is the moment of time, which is correspondent to reaching this maximum value of stress.

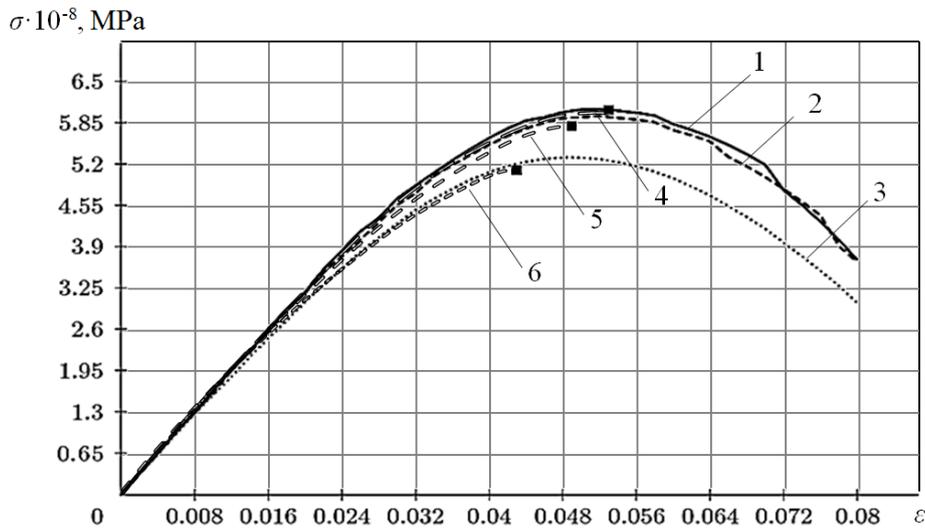
Relevant curves illustrating mutual relation between probabilistic model parameters and current experiment parameters in the form of temperature and strain were shown in Fig. 8.



**Figure 8.** Dependence of probabilities of fracture  $P_f$  (1) and recovery  $P_r$  (2) of bonds in material as of model parameters on the current parameters of the experiment, namely the strains (a) and temperatures (b)

Рис. 9 ілюструє порівняння даних розрахунку і експерименту за даними робіт [22, 26] для випадку високошвидкісного розтягання, тобто, з точки зору запропонованої моделі, докладання навантаження з підвищеними швидкостями деформації, але при незмінних температурних умовах.

Comparison of the data of simulation and experiment was shown in Fig. 9. These experimental data are described in [22, 26] for high strain rate tension, i.e. from the point of view of suggested modeling approach the test provided loading at increased strain rates but under constant temperature conditions.



**Figure 9.** Dependencies of stresses  $\sigma$  on the strain  $\varepsilon$  according to the results of simulation (1 – 3), and also experimental data [22, 26] (4 – 6): 1, 4 –  $\dot{\varepsilon} = 44,1 \text{ s}^{-1}$ ; 2, 5 –  $\dot{\varepsilon} = 4,41 \text{ s}^{-1}$ ; 3, 6 –  $\dot{\varepsilon} = 4,41 \cdot 10^{-1} \text{ s}^{-1}$ .

After considering the results of simulation according to the suggested model for investigated strain rates of up to  $44.1 \text{ s}^{-1}$ , stress-strain dependences  $\sigma(\varepsilon)$  have been calculated as functions  $\sigma_i = f[\tilde{C}_{ij}(D_{ij}), \varepsilon_j]$ , under condition that the strain rate  $\dot{\varepsilon}$  was practically constant during the simulated experiment. The relevant curves were shown in Fig. 9.

In curves depicted in Fig. 9, critical points were shown by black squares, which are correspondent to stresses and strains at fracture of the material and were obtained experimentally by other authors at investigated strain rates [22, 26]. The simulated curves in

Fig. 9 actually coincide with the experimental curves for relevant strain rates. Besides, curves, which were obtained for different strain rates, are clearly separated from each other. It means that the phenomenon of strengthening of the investigated composite is observed at increasing strain rate according to the results of experiment and also is successfully modeled in the provided simulation.

**Conclusions.** Reliable modeling of material damage processes is necessary for correct description of the stress-strain dependence under loading. For implementation of such modeling, a probabilistic model has been proposed in this article, which describes the relationship between the probability of occurrence of the physical processes of fracture and recovery of intermolecular and adhesive bonds between the structural particles of a composite and the factors influencing these processes, first of all, the value of strain, impact velocity and temperature.

Formulated probabilistic model was tested for simulating the case of tension of composite specimens at three different constant strain rates. Thus, it was shown that, by using the values of engineering properties of the material obtained under conditions of quasi-static tests, experimentally confirmed material hardening (increase of elasticity modulus and ultimate stress) at high strain rates is successfully predicted with the help of the proposed probabilistic model.

#### References

1. Astanin V.V., Shchegel A.A. Impact strain and fracture of hybrid composite materials – *Strength of Materials*, 2011, vol. 43, no. 6, pp. 615-627.
2. Frenkel D., Smit B. *Understanding molecular simulation: from algorithms to applications*. Computational Science Series, vol. 1, San Diego, Academic Press, 2 ed., 2002, 664 p.
3. Lennard-Jones J.E. On the determination of molecular fields – *Proc. R. Soc. Lond. A*, 1924, no. 106 (738), pp. 463-477.
4. Kaplan I.G. *Intermolecular interactions: physical picture, computational methods and model potentials* – Cornwall, Wiley, 2006, 380 p.
5. Rid R., Prausnits Dzh., SHervud T. *Svoystva gazov i zhidkостей* – St. Petersburg, Khimiya, 1982, 569 p. [In Russian].
6. Tager A.A., Askadskij A.A., ed. *Fiziko-khimiya polimerov* – Moscow, Nauchnyj mir, 2007, 573 p. [In Russian].
7. Allen M.P., Tildesley D.J. *Computer simulation of liquids* – Oxford, Uni. Press, 1990, 385 p.
8. ISO 527-5. *Kunststoffe. Bestimmung der Zugeigenschaften. Teil 5. Prüfbedingungen für unidirektional faserverstärkte Kunststoffverbundwerkstoffe*.
9. ISO 14126. *Faserverstärkte Kunststoffe. Bestimmung der Druckeigenschaften in der Laminebene*.
10. Mishnaevsky L., Brondsted P. Micromechanical modeling of fracture and fracture of unidirectional fiber reinforced composites: a review – *Comput. Mater. Sci.*, 2009, vol. 44, pp. 1351-1359.
11. Kim R.Y., Soni S.R. Experimental and analytical studies on the onset of delamination in laminated composites – *J. Compos. Mater.*, 1984, vol. 18, pp. 70-80.
12. Krevelen D., Nijenhuis K. *Properties of polymers: their correlation with chemical structure; their numerical estimation and prediction from additive group contributions* – N.Y., Elsevier, 2009, 1004 p.
13. Astanin V.V. Vliyanie skorosti deformatsii na predel tekuchesti metallov – *Mezhvuzovskij sbornik nauchnykh trudov "Sovershenstvovanie metodov rascheta i proektirovaniya konstruksij i sooruzhenij"* – KHar'kov, KHarDAZT, 1996, pp. 77-87. [In Russian].
14. Astanin V.V., Nadezhdin G.N., Stepanov G.V. Lokalizatsiya plasticheskoy deformatsii i razrusheniya alyuminiya i ego splavov pri udarnovolnovom nagruzhenii – *Dinamicheskaya prochnost' i treshhinostjokost' konstruksionnykh materialov*, Kyiv, Institut problem prochnosti AN USSR, 1986, pp. 260-266. [In Russian].
15. Astanin V.V., Romanchenko V.I. Prochnost' i szhimaemost' stekloplastika pri udare – *Mekhanika kompozitnykh materialov*, 1984, no. 4, pp. 731-734. [In Russian].
16. Astanin V.V., Sirenko A.A., Garipova G.EH. Razuprochnenie volokon bora pri vzaimodejstvii s matritsej – *Probl. Prochnosti*, 1993, no. 8, pp. 32-36. [In Russian].
17. Stepanov G.V., Astanin V.V. Opređenje soprotivleniya materiala sdvigu za frontom ploskoj udarnoj volny – *Probl. Prochnosti*, 1976, no. 4, pp. 94-98. [In Russian].
18. Nabarro B. *Theory of crystal dislocations* – Oxford, Uni. Press, 1967, 821 p.
19. Astanin V.V., Galiev SH.U., Ivashenko K.B. Osobennosti deformirovaniya i razrusheniya

aluminiumevykh pregrad pri vzaimodejstvii po normali so stal'nym udarnikom – Probl. Prochnosti, 1988, no. 12, pp. 52-58. [In Russian].

20. Astanin V.V. Deformirovanie i razrusheniya pregrad pri probivanii tsilindricheskim udarnikom – Dinamicheskaya prochnost' i treshhinostojkost' konstruksionnykh materialov, Kyiv, Institut problem prochnosti AN USSR, 1986, pp. 23-27. [In Russian].

21. Wang R. Introduction to orthogonal transforms: with applications in data processing and analysis – N.Y., Cambridge Uni. Press, 2012, 568 p.

22. Böhm R. Bruchmodebezogene Beschreibung des Degradationsverhaltens textilverstärkter Verbundwerkstoffe. Diss. akad. Grad. Dr.-Ing. Technische Universität Dresden, 2008, 123 p.

23. Gude M., Ebert C., Langkamp A., Hufenbach W. Characterisation and simulation of the strain rate dependent material behaviour of novel 3D textile reinforced composites – ECCM-13 European Conf. on Composite Materials, 2-5 June 2008, Stockholm, Sweden, Conf. Proc, 2008, pp. 1-15.

24. Simulia Abaqus Analysis. User Documentation – Providence, Dassault Systems, 2007, 651 p.

25. Lekhnitskij S.G. Teoriya uprugosti anizotropnogo tela – Moscow, Nauka, 1977, 2 nd ed., 416 p. [In Russian].

26. Hufenbach W., Gude M., Ebert C., Zscheige M., Hornig A. Strain rate dependent low velocity impact response of layerwise 3D-reinforced composite structures – Int. J. Impact Eng, 2011, no. 38, pp. 358-368.

### Список використаної літератури

1. Astanin, V.V. Impact strain and fracture of hybrid composite materials [Text] / V.V. Astanin, A.A. Shchegel // Strength of Materials. – 2011. – Vol. 43. – № 6. – P. 615 – 627.

2. Frenkel, D. Understanding Molecular Simulation: From Algorithms to Applications. Computational Science Series, vol. 1 [Text] / D. Frenkel, B. Smit. – San Diego: Academic Press. 2 ed., 2002. – 664 p.

3. Lennard-Jones, J.E. On the determination of molecular fields [Text] / J.E. Lennard-Jones // Proc. R. Soc. Lond. A. – 1924. – № 106 (738). – P. 463 – 477.

4. Kaplan, I.G. Intermolecular Interactions: Physical Picture, Computational Methods and Model Potentials [Text] / I.G. Kaplan. – Cornwall: Wiley, 2006. – 380 p.

5. Рид, Р. Свойства газов и жидкостей [Текст] / Р. Рид, Дж. Праусниц, Т. Шервуд. – Л.: Химия, 1982. – 569 с.

6. Тагер, А.А. Физико-химия полимеров [Текст] / А.А. Тагер. – А.А. Аскадский, ред. – М.: Научный мир, 2007. – 573 с.

7. Allen, M.P. Computer simulation of liquids [Text] / M.P. Allen, D.J. Tildesley. – Oxford: Oxford Uni. Press, 1990. – 385 p.

8. ISO 527-5. Kunststoffe – Bestimmung der Zugeigenschaften – Teil 5: Prüfbedingungen für unidirektional faserverstärkte Kunststoffverbundwerkstoffe.

9. ISO 14126. Faserverstärkte Kunststoffe – Bestimmung der Druckeigenschaften in der Laminebene.

10. Mishnaevsky, L. Micromechanical modeling of fracture and fracture of unidirectional fiber reinforced composites: a review [Text] / L. Mishnaevsky, P. Brondsted // Comput. Mater. Sci. – 2009. – Vol. 44. – P. 1351 – 1359.

11. Kim, R.Y. Experimental and analytical studies on the onset of delamination in laminated composites [Text] / R.Y. Kim, S.R. Soni // J. Compos. Mater. – 1984. – Vol. 18. – P. 70 – 80.

12. Krevelen, D. Properties of polymers: their correlation with chemical structure; their numerical estimation and prediction from additive group contributions [Text] / D. Krevelen, K. Nijenhuis. – N.Y.: Elsevier, 2009. – 1004 p.

13. Астанин, В.В. Влияние скорости деформации на предел текучести металлов [Текст] / В.В. Астанин // Межвузовский сборник научных трудов «Совершенствование методов расчета и проектирования конструкций и сооружений». – Харьков: ХарДАЗТ, 1996. – С. 77 – 87.

14. Астанин, В.В. Локализация пластической деформации и разрушения алюминия и его сплавов при ударноволновом нагружении [Текст] / В.В. Астанин, Г.Н. Надеждин, Г.В. Степанов // Динамическая прочность и трещиностойкость конструкционных материалов. – К.: Институт проблем прочности АН УССР, 1986. – С. 260 – 266.

15. Астанин, В.В. Прочность и сжимаемость стеклопластика при ударе [Текст] / В.В. Астанин, В.И. Романченко // Механика композитных материалов. – 1984. – № 4. – С. 731 – 734.

16. Астанин, В.В. Разупрочнение волокон бора при взаимодействии с матрицей [Текст] / В.В. Астанин, А.А. Сиренко, Г.Э. Гарипова // Пробл. прочности. – 1993. – № 8. – С. 32 – 36.

17. Степанов, Г.В. Определение сопротивления материала сдвигу за фронтом плоской ударной волны [Текст] / Г.В. Степанов, В.В. Астанин // Пробл. прочности. – 1976. – № 4. – С. 94 – 98.

18. Nabarro, V. Theory of crystal dislocations [Text] / V. Nabarro. – Oxford: Oxford University Press, 1967. – 821 p.

19. Астанин, В.В. Особенности деформирования и разрушения алюминиевых преград при

взаимодействии по нормали со стальным ударником [Текст] / В.В. Астанин, Ш.У. Галиев, К.Б. Иващенко // Пробл. прочности. – 1988. – № 12. – С. 52 – 58.

20. Астанин, В.В. Деформирование и разрушения преград при пробивании цилиндрическим ударником [Текст] / В.В. Астанин // Динамическая прочность и трещиностойкость конструкционных материалов. – К.: Институт проблем прочности АН УССР, 1986. – С. 23 – 27.

21. Wang, R. Introduction to orthogonal transforms: with applications in data processing and analysis [Text] / R. Wang. – N.Y.: Cambridge University Press, 2012. – 568 p.

22. Böhm, R. Bruchmodebezogene Beschreibung des Degradationsverhaltens textilverstärkter Verbundwerkstoffe [Text] / R. Böhm // Diss. akad. Grad. Dr.-Ing. – Technische Universität Dresden. – 2008. – 123 p.

23. Gude, M. Characterisation and simulation of the strain rate dependent material behaviour of novel 3D textile reinforced composites [Text] / M. Gude, C. Ebert, A. Langkamp, W. Hufenbach // ECCM-13: European Conf. on Composite Materials, 2 – 5 June 2008, Stockholm, Sweden: Conf. Proc. – 2008. – P. 1 – 15.

24. Simulia Abaqus Analysis. User Documentation / Simulia. – Providence: Dassault Systems, 2007. – 651 p.

25. Лехницкий, С.Г. Теория упругости анизотропного тела [Текст] / С.Г. Лехницкий. – М.: Наука, 1977. – 2-е изд. – 416 с.

26. Hufenbach, W. Strain rate dependent low velocity impact response of layerwise 3D-reinforced composite structures [Text] / W. Hufenbach, M. Gude, C. Ebert, M. Zschehyge, A. Hornig // Int. J. Impact Eng. – 2011. – № 38. – P. 358 – 368.

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## **ЙМОВІРНІСНЕ МОДЕЛЮВАННЯ ФІЗИЧНИХ ПРОЦЕСІВ ПОШКОДЖЕННЯ ПЛАСТИН ІЗ ВОЛОКОННОЗМІЦНЕНИХ КОМПОЗИТІВ ПРИ ДИНАМІЧНОМУ НАВАНТАЖЕННІ**

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***Резюме.** На основі аналізу фізичних процесів пошкодження внутрішньої структури композитів і розглянутої раніше феноменологічної моделі удару в діапазоні швидкостей зіткнення від 20 до 1500 м/с запропоновано ймовірнісну модель міцності композиційного матеріалу з урахуванням еволюції пошкодження зв'язків між його структурними елементами. Проведено порівняння результатів розрахунку з отриманими іншими авторами експериментальними даними для випадку розтягу волоконнозміцненого матеріалу при швидкостях деформації від 0,441 до 44,1 с<sup>-1</sup>.*

***Ключові слова:** волоконнозміцнені композити, еволюція пошкодження, ймовірнісне моделювання, високошвидкісний удар, швидкість деформації.*

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