УДК 637.022

Математичне моделювання процесу розділення суспензії на фільтрі із самоочисним фільтрувальним елементом

Марія Шинкарик, Олег Кравець
Тернопільський національний технічний університет імені І. Пулюя

Вступ. Метою роботи була побудова математичної моделі процесу фільтрування суспензії на фільтрі із самоочисним фільтрувальним елементом, що виконаний у вигляді циліндричної пружини стиску.

Матеріали і методи.

Дослідження процесу фільтрування проводили на фільтрі із самоочисним фільтрувальним елементом. В якості дослідної суспензії використовували молочну сироватку, отриману при виробництві сиру кисломолочного. Концентрацію дисперсної фази в сироватці визначали шляхом центрифугування проб з подальшим висушуванням осаду в сушильній шафі.

Результати і обговорення. Отримана математична модель ґрунтується на моделі процесу фільтрування із закупорюванням кожної пори окремою частинкою.

В моделі враховується, що не всі частинки дисперсної фази, розмір яких перевищує ширину фільтрувальних отворів, будуть їх закупорювати, а лише їх частка, що прямо пропорційна відношенню площі живого перерізу до загальної площі фільтрувальної поверхні.

Математична модель дозволяє визначати тривалість процесу фільтрування виходячи із об’єму суспензії та встановлювати раціональний період між регенераціями самоочисного фільтрувального елемента.

Порівняння параметрів отриманих шляхом математичного моделювання із реальним процесом фільтрування молочної сироватки свідчить, що математична модель адекватно відображає процес розділення суспензії на фільтрі із самоочисним фільтрувальним елементом при об’ємі фільтрату від 0 до 5 м^3 на 1 м^2 фільтрувальної поверхні. Середнє відносне відхилення результатів отриманих з допомогою математичної моделі від експерименту становить 11 %.

Висновки. Математична модель може бути застосована при розрахунку параметрів процесу фільтрування суспензії на фільтрі із самоочисним фільтрувальним елементом.

Ключові слова: фільтрування, суспензія, пори, закупорювання, капіляри, регенерація.

Математическое моделирование процесса разделения суспензии на фильтре с самоочищающимся фильтрующим элементом

Мария Шинкарик, Олег Кравец
Тернопольский национальный технический университет имени И. Пулюя

Введение. Целью работы было построение математической модели процесса фильтрации суспензии на фильтре с самоочищающимся фильтрующем элементом, который выполнен в виде цилиндрической пружины сжатия.

Материалы и методы. Исследование процесса фильтрации проводили на фильтре с самоочищающимся фильтрующем элементом. В качестве исследовательской суспензии использовали молочную сыворотку, полученную при производстве творога. Концентрацию дисперсной фазы в сыворотке определяли путем центрифугирования проб с последующим высушиванием осадка в сушильном шкафу.

Результаты и обсуждение. Полученная математическая модель основывается на модели процесса фильтрации с закупориванием каждой поры отдельной частицей.

В модели учитывается, что не все частицы дисперсной фазы, размер которых превышает ширину фильтровальных отверстий, будут их закупоривать, а только их часть, прямо пропорциональна отношению площади живого сечения к общей площади фильтрующей поверхности.

Математическая модель позволяет определять продолжительность процесса фильтрации исходя из объема суспензии и устанавливать оптимальный период между регенерациями самоочищающегося фильтрующего элемента.

Сравнение параметров полученных путем математического моделирования с реальным процессом фильтрования молочной сыворотки свидетельствует, что математическая модель адекватно отражает процесс разделения суспензии на фильтре с самоочищающимся фильтрующим элементом при объеме фильтрата от 0 до 5 м^3 на 1 м^2 фильтрующей поверхности. Среднее относительное отклонение результатов полученных с помощью математической модели от эксперимента составляет 11%.

Выводы. Математическая модель может быть применена при расчете параметров процесса фильтрования суспензии на фильтре с самоочищающимся фильтрующим элементом.

Ключевые слова: фильтрование, суспензия, времени, укупорки, капилляры, регенерация.

Mathematical modelling of the separation of suspension process on the filter with selt-purifier filter element

Mariia Shynkaryk, Oleh Kravets
Ternopil I. Puliui National Technical University

Introduction. The aim of the work was to build a mathematical model of the process of suspension filtering on the filter with self-purifier filter element that is designed as a cylindrical compression spring.

Materials and methods. Research of the filtering process was performed on the filter with self-purifier filter element. As a studied suspension the milk whey was used, obtained in the production of cottage cheese. The concentration of the dispersed phase in the milk whey was defined by centrifugation of samples followed by further drying sludge in a drying oven.

Results and discussion. The gained mathematical model is based on the model of the filtering process with clogging each pore with the individual particle.

The model takes into account that not all of the dispersed phase particles that are larger than the width of the filter holes, will clog them, but only their particle that is directly proportional to the ratio of living area to the total area of the filter surface.

The mathematical model allows to determine the length of the filtering process based on the volume of suspension, and to set the rational period between regenerations of self-purifier filter element.

Comparing the parameters obtained by mathematical modelling with the real process of filtering milk whey indicates that the mathematical model adequately reflects the separation process of suspension on the filter with self-purifier filter element with the volume of filtrate from 0 to 5 m^3 per 1 m^2 filter surface - the average relative deviation of the results obtained with the help of the mathematical model of the experiment is 11%.

Conclusions. The mathematical model can be applied in calculating parameters of the process of suspension filtering on the filter with self-purifier filter element.

Keywords: filtering, suspension, pores, clogging, capillaries, regeneration.

Mathematical modelling of the separation of suspension process on the filter with selt-purifier filter element

Mariia Shynkaryk, Oleh Kravets
Ternopil I. Puliui National Technical University

Introduction

Regimes of suspensions filtering process are due to the following main factors: properties of the suspension, properties of the filtration surface and the design properties of the filter. Regarding filtration properties of the sludge, the impact of both properties of dispersed environment [1-4] and dispersion [5, 6] should be noted.

Filtering through a layer of sludge is described by Darcy [7]:

$$W = \frac{1}{\mu r_0} \cdot \frac{\partial P}{\partial h}, \qquad (1)$$

where W – speed of filtering, m/sec;
r_0 – filtering resistivity, m^{-2};
μ – dynamic viscosity of suspension, Pa·sec;
P – pressure, Pa;
h – height of sludge layer, m.

In the mathematical description of the filtering process the compressibility of the sludge and continuity equation of the solid and liquid phase are also taken into account [8]:

$$\frac{\partial W}{\partial h} = -\frac{\partial \varepsilon}{\partial \tau}, \qquad (2)$$

where ε – porosity of the sludge layer;
τ – duration of filtering, sec.

For consideration of sludge's compressibility the law of compression can be used under which the infinitely small change in sludge's porosity is directly proportional to the infinitely small change of pressure:

$$G = \frac{\partial e}{\partial P}, \qquad (3)$$

where G – compressibility module of sludge, Pa;
e – coefficient of porosity.

Solving this equation considering filtration and compression properties of sludge is given in the work [8]

Also [8] the work highlights the need for representation filtering equation as follows:

$$W - e\sigma = \frac{1}{\mu r_0} \cdot \frac{\partial P}{\partial h}, \tag{4}$$

where σ – speed of the solid phase movement, m/sec.

In the work [1] it is proposed to consider the dependence of the resistivity to filtering of the pressure filtering.

In the work [9] non-Newton properties of the dispersion phase, considering Rut's equation are proposed:

$$\frac{dq}{d\tau} = \left[\frac{\Delta R}{kr_0 x(V - V_e)}\right]^{\frac{1}{n}}, \tag{5}$$

where q – specific volume of the filtrate obtained of the area unit of filter surface, m^3/m^2;
ΔR – hydraulic resistance to the layer of sludge and filter surface, m^{-1};
k – consistency index;
n – stream index of filtrate;
x – ratio of volumes of the obtained sludge and filtrate;
V – volume of filtrate, m^3;
V_e – equivalent volume of filtrate, during the passage of which a sludge layer with resistance to filtering that equals resistance to filter surface may be formed, m^3.

Secure features of the filtering process are considered in the monograph [1], in particular filtration at a constant speed, constant pressure of the process, etc.

Common to all these works is filtration with a sludge formation on the filter surface and change of the layer of sludge's height. However, structures of filtering machines the height of the sludge layer of which is minimal and unchanged throughout the whole process are known. This includes filtering centrifuges with continuous diversion of sludge and filters with self-purifier filter elements [7]. In this case, resistance to filtering, which creates a layer of sludge, can be considered as resistance of the filter surface, and filtration process as such that proceeds with clogging pores.

Filtering theory [7] in this case considers the following cases: filtering with clogging every pore with a separate solid particle (complete clogging of pores) filtering with a gradual clogging of every pore with many solid particles.

Mathematical model of filtering every pore process with a separate solid particle is presented in the publication [7]. The author proposes to consider that on the filtration surface with area of 1 m^2 is N identical cylindrical capillaries of radius r and height, corresponding to the height of the sludge's layer. The initial speed of filtration is proposed to be determined of the expression:

$$W_0 = AN, \tag{6}$$

where W_0 – initial speed of filtering, m/sec;
N – number of capillaries;

A – volume of filtrate that passes through the capillaries per second, m^3/sec, is found of Hagen–Poiseuille equation:

$$A = \frac{\pi r^4 \Delta P}{8\mu h}. \tag{7}$$

After passing the filtrate in the amount V the number of the clogged capillaries is the following:

$$N_c = nV, \tag{8}$$

where N_c – number of the clogged capillaries;
n – amount of the particles of a dispersed phase in one m^3 of suspension that clog capillaries, $1/m^3$.

In these conditions, the dependence of filtering speed of the volume of filtrate will be described by the equation:

$$W=A(N - nV). \tag{9}$$

However, the formulation of the problem in this form does not allow taking into account design features of filter and suspension.

The aim of the work was to build a mathematical model of the process of suspension filtering with every pore clogging of the self-purifier filter element with one solid particle in the absence of the sludge layer.

Materials and methods

The object of the research was the process of suspension filtering and theoretical description of this process. Theoretical analysis was performed for the installation with self-purifier filter element.

This refers to the self-purifier filter element that is designed as a cylindrical spring of compression, the size of the gap between the turns of which corresponds to the size of the smallest particle of the dispersed phase that should be impeded [10].

Experimental instalment with a cylindrical filter element consisted of a cylindrical body 1 (Fig. 1), nozzles 2 and 3, supply and removal of whey respectively, guide glass 4 with a screw 5, tube to remove sludge 6, filter element 7, ring 8, electromagnet 9, shaft 10, sludge tank 11 and measuring tank 12. Material of the filter element is stainless steel; the gap between the turns of the filter element - 0.9 mm, the proportion of living cut - 38%.

As the pilot suspension the milk whey was used, obtained in the production of cottage cheese in a periodic way (fat content in the finished product - 9%) using cheese-producing baths of brand VS-5000.

Firstly a concentration of dispersed particles of protein in whey was defined. For this case sampling of whey with volume of 500 ml was made every 3 minutes during the whole time of pouring the whey out of the cheese-producing bath. Then all samples were poured into one container, mixed and 12 portions of 5 ml each were selected of the total volume of whey. Then four portions of whey were poured into four tubes, closed with rubber stoppers and placed into a centrifuge of mark OPN-12 with plugs to center. The whey was centrifuged for 5 min at speed of 6000 rev/min (centrifugation duration was measured from the moment of gaining rotational speed - 6000 rev/min). After centrifugation the tubes were removed from the centrifuge, opened and the liquid phase was poured so that only the sludge remained in the tube. The obtained sludge was dried in the oven for 60 minutes at the temperature of 105° C. The dried sludge was weighed. The concentration of dispersed particles of protein in the whey was defined as the ratio of dry sludge to the volume of whey of which it was obtained (2·10-5 m3). The experiment was repeated three times. The concentration of dispersed particles of protein in the whey was 3.0 kg/m^2.

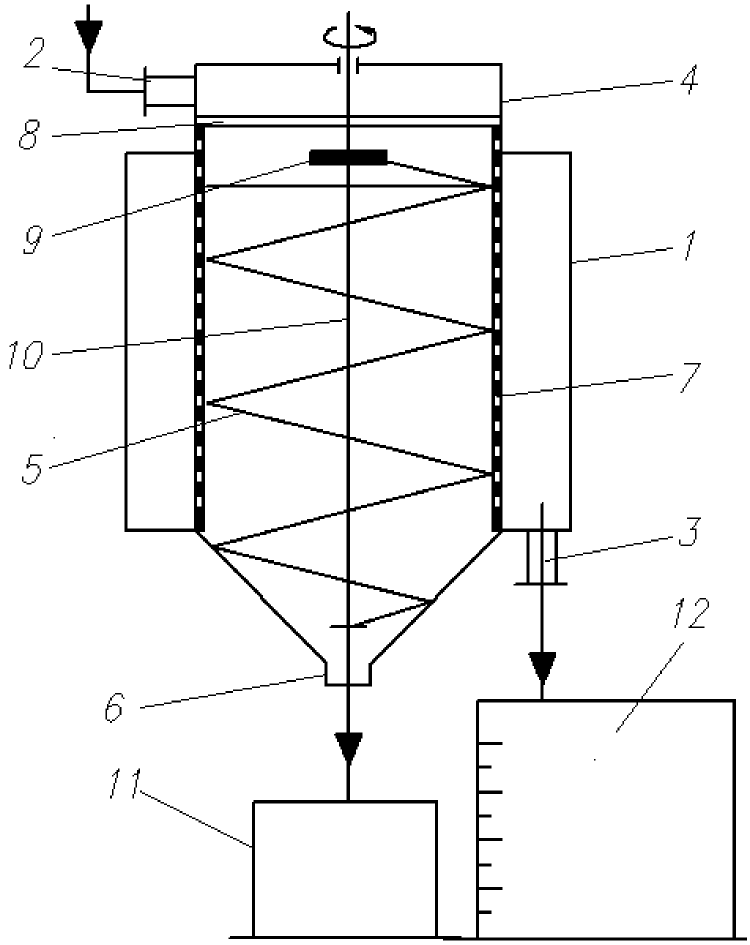

Figure 1. Experimental installation with self-purifier filter element
1-body; 2-3, respectively tubes of supply and removal of whey; 4-guide glass; 5-screw; 6-nozzle to remove sludge; 7 self-purifier filter element; 8-ring; 4; 9- electromagnet; 10-shaft; 11-sludge tank; 12-measuring filtrate tank.

Further purification of the whey was performed. Milk whey was given to the experimental installation directly of cheese-producing bath (without pump) under the pressure of 3.0 kPa. The whey through the pipe 2 was tangentially given to the glass 4. From the glass it passed through the filter element 7. The ring 8 with periodic switch of electromagnet 9 was moved down along the axis of the shaft 10 and thus compressed filter element 7 that ensured its regeneration. The duration of the regeneration was 1 sec. The sludge that remained on the filter element was transported with screw 5 into the bottom of the body 1, where it was removed periodically through the pipe 6 into the tank 11. The filtrate was removed of the filter through the pipe 3 into the measuring tank 12.

Screw rotation rate was 9 rev/min. During the experiment every 10 sec the amount of the sludge was fixed.

After filtering whey samples were collected out of the measuring tank 12, 5 ml each, and according to the methodology described above, the concentration of the dispersed phase in the filtrate was defined. It was 1.6 kg/m^3.

Results and discussion

In the mathematical model developing the following assumptions were made: during the accumulation of sludge in the gaps between the turns of the spring the capillaries were formed, diameter of which is equal to the distance between the coils of the spring; the height of the capillary is equal to the wall thickness of the filter element (diameter of the coil spring); dispersed phase particles are uniformly distributed over the entire area of the filter surface.

It was believed that the number of capillaries N is proportional to the living cut area of filter surface:

$$N = \frac{S_l}{\pi r_c^2}, \tag{10}$$

Where S_l – living cut area of filter surface, m^2;
r_c – capillary radius, m.

Then we assume that not all of the dispersed phase particles that are larger than the diameter of capillaries will clog them, but only their particle that is directly proportional to the ratio of living cut area to the total area of the filter surface. The remaining particles will be laid on coils of the spring and will be transported by screw. Then n will be:

$$n = n_0 \frac{S_l}{S}, \tag{11}$$

where n –number of the dispersed phase particles in 1 m^3 of suspension that are larger than pore diameter, 1/m^3;
S – filtering surface area, m^2.

We change in equation (9) W to dV/dτ:

$$\frac{dV}{d\tau} = A(N - nV). \tag{12}$$

We divide variables:

$$\frac{dV}{A(N-nV)} = d\tau, \tag{13}$$

$AN = W_0$, mark $A \cdot n = m$.

After integration of equation (13) from 0 to V and from 0 to τ, with boundary conditions V = 0 and τ = τ0, we obtain an equation for the duration of filtering:

$$\tau = \tau_0 - \frac{1}{A \cdot n} \ln\left(\frac{N - nV}{N}\right), \tag{14}$$

where τ_0 – duration of the filtering with a clean filter surface (none of capillaries is clogged), sec.

Verification of the mathematical model for adequacy was performed on the example of the process of milk whey filtering. Sequence of checking was as follows: experimental study of the milk whey filtering process in production conditions was made; mathematical modelling of the milk whey filtering process was performed; data obtained in the experimental way with the appropriate calculations were compared; relative deviation of the mathematical model from the real process of filtering was defined according to the formula:

$$\Omega = \frac{\sum_{i=1}^{j} (|X_i - Y_i|)}{\sum_{i=1}^{j} X_i} \cdot 100\%, \tag{15}$$

where Ω – relative deviation, %;

$X_{i.}$ – experimental value of filtering duration to filtrate volume V_i, sec;

Y_i – calculated value of filtering duration (according to the formula (14)) for the filtrate volume V_i, sec;

j – number of measurements, j = 10.

Mathematical modelling was carried out in the following sequence:

- number of capillaries was defined according to the formula (10);
- average radius of particles larger than the diameter of the pores of the expression was defined:

$$r_{cep} = \frac{\sum_{i=1}^{3} (r_i g_i)}{\sum_{i=1}^{n} g_i}, \tag{16}$$

Where r_i – average radius of i-fraction of protein particles larger than the width of the filter surface gap;

g_i – share of i-fraction protein particles, %.

- average weight of one protein particle was found:

$$m_{p.av.} = \frac{3}{4}\pi(r_{c.av.})^3\rho_p, \tag{17}$$

where $m_{p.av.}$ – average weight of protein particles larger than the diameter of the pore, kg;
$r_{c.av.}$ – average radius of capillaries, m;
ρ_p – density protein, kg/m^3.

- number of particles in 1 m^3 of suspension larger than the size of the pore was defined with the help of the formula:

$$n_0 = \frac{m_\Sigma}{m_{p.av.}}, \tag{18}$$

where m_Σ – total mass of particles larger than the diameter of the pore, kg.

- number of the dispersed phase particles in 1 m^3 of suspension that clog the pores according to the formula was defined (11);
- volume of the filtrate that passes through the capillary in one second according to the equation was found (7);
- length of the filter with a clean filtration surface was defined:

$$\tau_0 = \frac{V}{FW_0}, \tag{19}$$

where F – area of filter surface, m^2.

- calculation of the duration of the filtering process at V = 0, 1, 2, 3 ... 10 m^3 according to the formula (14) was performed.

Comparison of the data obtained by mathematical modelling with the real filtering process of milk whey (figure 2) shows that the mathematical model adequately reflects the process of suspension separation on the filter with self-purifier filter element with the specific volume of filtrate up to 5 m^3/m^2 - the average relative deviation of the results obtained with the help of the mathematical model of the experiment is 11%. With further increase of the specific volume of the filtrate the relative deviation of the mathematical model increases.

As the concentration of dispersed particles in suspension remains stable and the number of unclogged filter openings is inversely proportional to the volume of filtrate, at a constant speed of filtering the process of clogging is eventually accelerated. So, the lines showing the filter length dependence on the volume of filtrate are of non-linear character (figure 2).

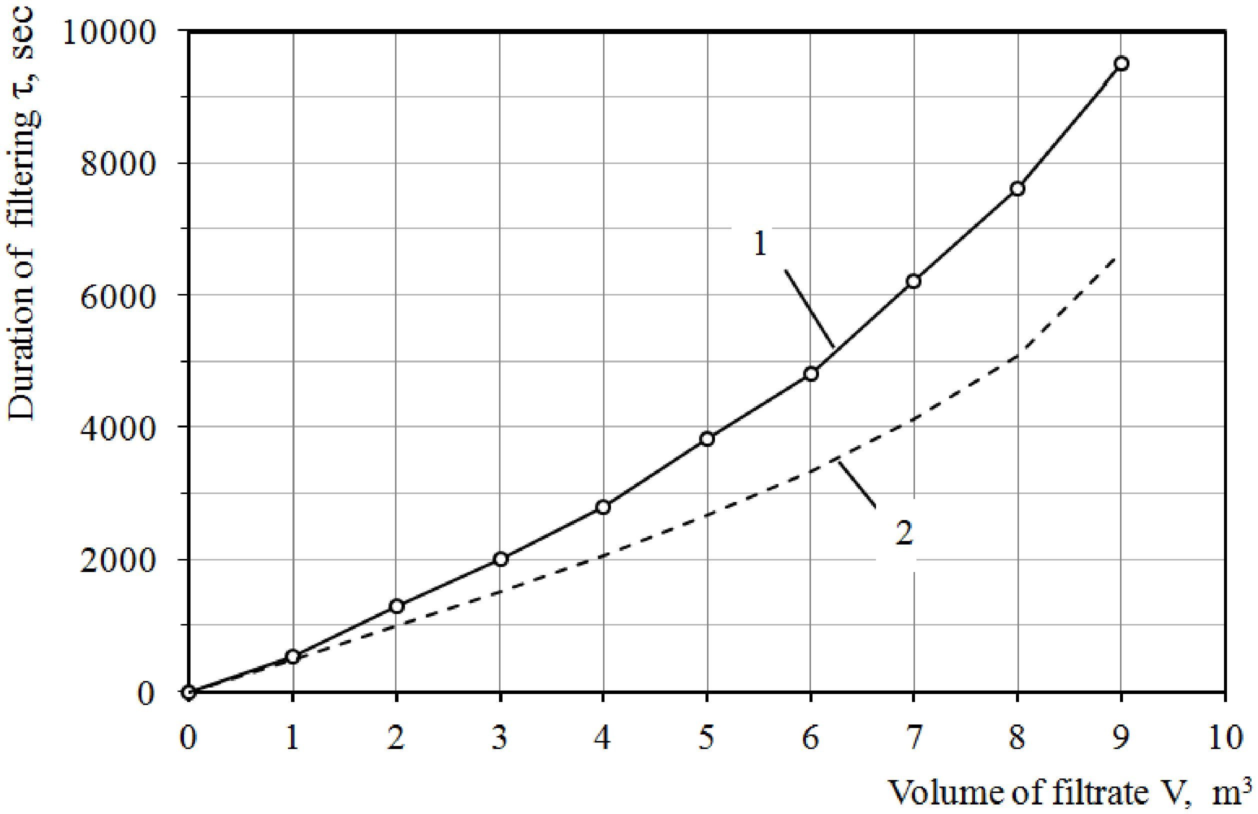

Figure 2. Dependence of filtering milk whey duration filtrate on the volume of filtrate for 1 m^2 of filter surface:

1 – experimental data; 2 – calculation according to the formula (14).

Conclusions.

The proposed mathematical model of suspension filtering process with every pore clogging of self-purifier filter element with a separate particle at absence of sludge layer allows to predict the duration of filtering depending on the volume of the obtained filtrate and to set a rational value of regeneration period of the filter element.

The mathematical model adequately reflects the separation process of suspension on the filter with self-purifier filter element with at the volume of filtrate from 0 to 5 m^3 per 1 m^2 of filter surface - average relative deviation of results obtained with the help of the mathematical model of the experiment is 11%.

It can be used in calculation of the regeneration process of self-purifier filter element and when designing new filter installations.

References

1. Kravets O.I., Shinkarik M.N. (2013), Vliyanie reologicheskikh kharakteristik syrnoy pyli na protsess fil'trovaniya molochnoy syvorotki, *Izvestiya vuzov. Pishchevaya tekhnologiya*, 5-6, pp. 71-74.

2. Tao D., Groppo J.G., Parekh B.K. (2000), Effects of vacuum filtration parameters on ultrafine coal dewatering, *Coal Preparation*, 21, pp. 315 - 335.

3. Kuzrnin G. A., Soboleva O. N. (2004), Subgrid modeling of filtration in a porous medium with multiscale log-stable permeability, *Monte Carlo Methods and Applcations*, 10 (3-4), pp. 369-376.

4. Gong Y.-W., Zhang H.-X., Cheng X.-N. (2012), Treatment of dairy wastewater by two-stage membrane operation with ultrafiltration and nanofiltration, *Water Science & Technology*, 65(5), pp. 915-919

5. Turan M. (2004), Influence of filtration conditions on the performance of nanofiltration and reverse osmosis membranes in dairy wastewater treatment, *Desalination*, 170, pp. 83-90.

6. Anlauf H. (2008), Liquid separation through cake filtration, *F&S Filtrieren und Separieren: Global Guide of the Filtration and Separation Industry*, 5, pp. 108-114.

7. Zhuzhikov V.A. (1980), *Fil'trovanie, Teoriya i praktika razdeleniya suspenziy*, Khimiya, Moscow.

8. Vorob'ev E.I., Shinkarik M.N. (1988), Matematicheskaya model' razdeleniya zhidkoy i tverdoy faz otzhimaniem, *Zhurnal prikladnoy khimii*, 22(2), 226p.

9. Balashov V. A., Efimov M. V. (2010), Uravnenie Rutsa dlya razdeleniya suspenziy s psevdoplasticheskoy dispersionnoy sredoy, *Izvestiya volgogradskogo gosudarstvennogo tekhnicheskogo universiteta*, 1(3), pp. 79-80.

10. Kravets O.I., Shynkaryk M.M. (2013), Reheneratsiia filtruvalnoi poverkhni pry ochysttsi molochnoi syrovatky, *Ukrainian Food Journal*, 4, pp. 555-561.