Вісник Тернопільського національного технічного університету
Scientific Journal of the Ternopil National Technical University
2016, № 1 (81)
ISSN 1727-7108. Web: visnyk.tntu.edu.ua

MATHEMATICAL MODELING. MATHEMATICS

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ. МАТЕМАТИКА

UDC 627.324.2/3:532.72

FINITE ELEMENT METHOD IN SPATIAL FILTRATION CONSOLIDATION PROBLEM WITH THIN SEMI-PERMEABLE INCLUSIONS UNDER THE INFLUENCE OF THE HEAT AND SALT TRANSFER

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Summary. *Mathematical model of filtration consolidation of soils, taking into account available there thin semi-permeable inclusions in three-dimensional case, has been constructed. The finite element method was used for numerical solution of three-dimensional problem. A number of numerical experiments has been done and the influence of the heat and salt transfer and semi-permeable inclusions on the distribution of the pressures in soil massifs has been shown.*

Key words: *filtration consolidation, matching conditions, semi-permeable inclusion, finite element method.*

Received 11.01.16

Introduction. Soil massifs can contain heterogeneity under natural conditions [7, 15, 16]. Semi-permeable inclusions, through properties of the materials, from which they are composed, are free to pass water and partly to pass various chemicals dissolved in water. This kind of inclusions has properties of semi-permeable membranes [17].

Availability of semi-permeable properties in the clays is explained by the phenomenon of chemical osmosis, in other words, by the dissolvent flow through semi-permeable partition, which is free to pass molecules of dissolvent and blocks molecules of dissolved substance [6]. Osmosis appears at micro and macro levels in soils. Availability of osmosis flow is like filtration anomaly, so far as osmotic flow, depending on direction, can speed up diffusion and filtration or prevent them.

The phenomena of internal and external osmosis, under the influence of which double electrical layer of particles, that serves as a semi-permeable membrane and correspondingly has their properties, were investigated in the article [13]. Due to this, there is the necessity of forecast amendment of engineering structures based on the soils with semi-permeable inclusions.

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The actuality of the research of consolidation processes in multivendor environment is explained by the necessity of defining stability of industrial, civil, hydro-technical structures built on heterogeneous soil layers [10].

Consolidation of heterogeneous soil foundation during vertical drainage establishment was investigated in [11]. Mathematical model was built in three-dimensional case. The solution of corresponding boundary problem was shown in the form of combination of Bessel functions.

Fractional differential mathematical model for investigation of locally unbalanced geo-migration processes, including chemical osmosis and ultra filtration, and mathematical modeling for consolidation dynamics of double-layer geoporous massif located on impenetrable base non-equilibrium in time, were constructed in [1, 2].

Works of the following scholars as I. Serhiyenko, V. Skopetskyy, V. Deyneka, A. Vlasyuk, Th. J.S. Keijzer, M. Petryk were dedicated to processes of heterogeneous porous medium [7, 14 – 16, 17, 18]. Particularly, mathematical models for filtration soil consolidation under the influence of the heat and salt transfer with the availability of thin inclusions which have characteristics of semi-permeable membranes were constructed in [4]. However, only one-dimensional case of problem was examined there.

There were a great number of damages because of non-uniform settings of big buildings built in the first half of the last century, when only general theoretical principles of filtration consolidation of clay soils were known. For example, there was a collapse of Transcona elevator in Canada, which was located on lake clay deposits with a depth of 9 m [8].

Nowadays, the whole range of examples emphasizes the actuality of investigations of filtration consolidation processes, particularly taking into consideration salt transfer under anisothermic conditions, – while building and operation of Rivne, Zaporizhzhya, Balakiv atomic power stations, the necessity arose for solving problems, concerning non-uniform soil deformation of bases of constructions. In particular, while building Balakiv atomic power station, counter-loads of thousand tons (hinged vessels filled with water), the weight of which had to be always corrected, were used to avoid big slopes of reactor blocks [12].

The special attention should be paid to three-dimensional case. It shows the influence of different factors the most completely and equally, and study of three-dimensional case gives a possibility to take into consideration the area of consolidation, and to set a location of thin clay semi-permeable inclusions in sand soils more precisely.

Research objective is to investigate processes of consolidation of soil with available there semi-permeable inclusions in three-dimensional case and the influence of the heat and salt transfer on the given processes in soil.

Problem statement and its mathematical model. Considering the given area Ω, consisting of homogeneous soil and thin semi-permeable inclusions, without loss of generality, we will suppose that such inclusion is one (γ^* Fig. 1).

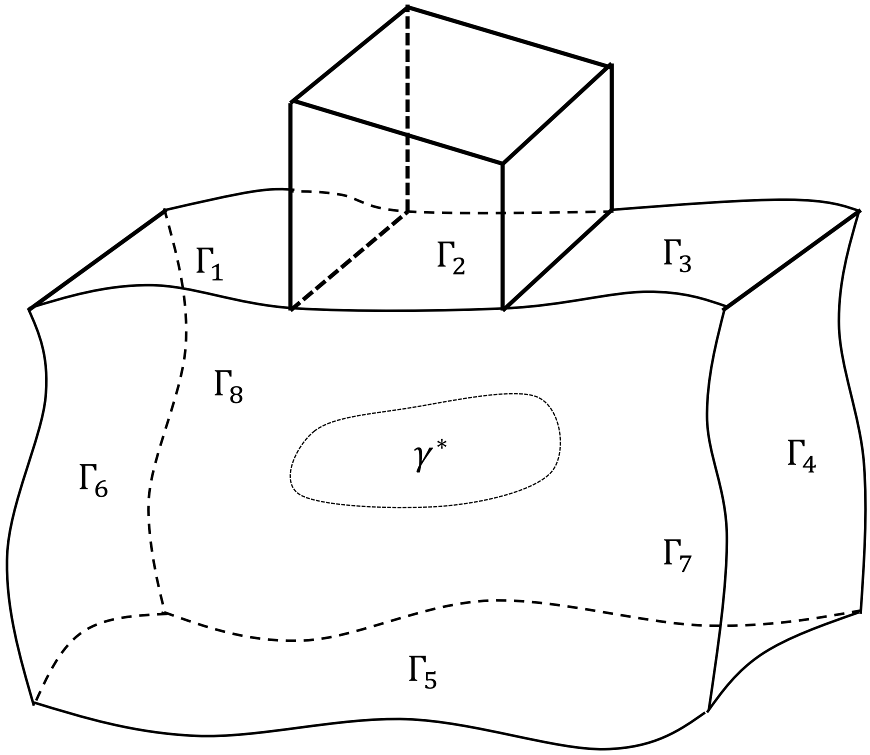

Figure 1. Spatial area of the soil with semi-permeable inclusion

Mathematical model of three-dimensional case of filtration consolidation process taking into account the influence of salt transfer in nonisothermal condition will be described by the following boundary value problem [3, 4]:

$$\frac{(1+e)(1+2\xi)}{3\gamma a}[div(\boldsymbol{K}\nabla h-\boldsymbol{v}\nabla c-\boldsymbol{\mu}\nabla T)]=\frac{\partial h}{\partial t},\ (\mathrm{x,y,z})\in\Omega,\ \mathrm{t}>0, \tag{1}$$

$$div(\boldsymbol{D}\nabla c+\boldsymbol{D_T}\nabla T)-(\boldsymbol{u},\ \nabla c)-\gamma_1(c-C_m)=\sigma\frac{\partial c}{\partial t},(\mathrm{x,y,z})\in\Omega,\mathrm{t}>0, \tag{2}$$

$$div(\boldsymbol{\lambda}(c)\nabla T)-\rho c_\rho(\boldsymbol{u},\nabla T)=c_T\frac{\partial T}{\partial t},(\mathrm{x,y,z})\in\Omega,\mathrm{t}>0, \tag{3}$$

$$\boldsymbol{u}=-\boldsymbol{K}(c,T,e)\nabla h+\boldsymbol{v}(c)\nabla c+\boldsymbol{\mu}\nabla T, \tag{4}$$

$$(\boldsymbol{u},\boldsymbol{n})|_{(x,y)\in\Gamma_2\cup\Gamma_4\cup\Gamma_5\cup\Gamma_6\cup\Gamma_7\cup\Gamma_8}=0, \tag{5}$$

$$h|_{(x,y)\in\Gamma_1}=H_1,\ h|_{(x,y)\in\Gamma_3}=H_2, \tag{6}$$

$$(\boldsymbol{D}\nabla c+\boldsymbol{D_T}\nabla T-\boldsymbol{u}c,\boldsymbol{n})|_{(x,y)\in\Gamma_2\cup\Gamma_4\cup\Gamma_5\cup\Gamma_6\cup\Gamma_7\cup\Gamma_8}=0, \tag{7}$$

$$c|_{(x,y)\in\Gamma_1}=c_1,\ c|_{(x,y)\in\Gamma_3}=c_2, \tag{8}$$

$$(\lambda(c)\nabla T-\rho c_{\rho}\boldsymbol{u}T,\boldsymbol{n})|_{(x,y)\in\Gamma_2\cup\Gamma_4\cup\Gamma_5\cup\Gamma_6\cup\Gamma_7\cup\Gamma_8}=0, \quad (9)$$

$$T|_{(x,y)\in\Gamma_1}=T_1,\ T|_{(x,y)\in\Gamma_3}=T_2, \quad (10)$$

$$(\boldsymbol{u},\boldsymbol{n})^{\pm}_{\gamma^*}=-\frac{k^{\gamma^*}}{d}\left(h^+-h^-\right)+\frac{\nu^{\gamma^*}}{d}\left(c^+-c^-\right)+\frac{\mu^{\gamma^*}}{d}\left(T^+-T^-\right),(x,y)\in\gamma^*,t>0, \quad (11)$$

$$(\boldsymbol{q_c},\boldsymbol{n})^{\pm}_{\gamma^*}=(1-\alpha)\left((\boldsymbol{u},\boldsymbol{n})^{\pm}_{\gamma^*}-\frac{D^{\gamma^*}}{d}\left(c^+-c^-\right)-\frac{D_T^{\gamma^*}}{d}\left(T^+-T^-\right)\right),(x,y)\in\gamma^*,t>0, \quad (12)$$

$$(\boldsymbol{q_T},\boldsymbol{n})^{\pm}_{\gamma^*}=\rho c_{\rho}(u,n)^{\pm}_{\gamma^*}T^{\pm}-\frac{\lambda^{\gamma^*}}{d}\left(T^+-T^-\right),(x,y)\in\gamma^*,t>0, \quad (13)$$

$$\left[(\boldsymbol{u},\boldsymbol{n})\right]|_{\gamma^*}=0,\ \left[(\boldsymbol{q_c},\boldsymbol{n})\right]|_{\gamma^*}=0,\ \left[(\boldsymbol{q_T},\boldsymbol{n})\right]|_{\gamma^*}=0,t>0, \quad (14)$$

$$\boldsymbol{q_c}=\boldsymbol{u}c-\boldsymbol{D}\nabla c-\boldsymbol{D_T}\nabla T,\ \boldsymbol{q_T}=\rho c_{\rho}\boldsymbol{u}T-\boldsymbol{\lambda}(c)\nabla T, \quad (15)$$

$$h(x,y,z,0)=h_0(x,y,z),\ c(x,y,z,0)=c_0(x,y,z),\ T(x,y,z,0)=T_0(x,y,z), \quad (16)$$
$$(x,y,x)\in\overline{\Omega},$$

where h – pressure; c – concentration; T – temperature; $\boldsymbol{K}=\boldsymbol{K}(c,T,e)$ – filtration coefficient; $\boldsymbol{\nu}$ – chemical osmosis coefficient; $\boldsymbol{\mu}$ – thermal osmosis coefficient; γ – specific gravity of pore liquid; e – coefficient of soil porosity; ξ – coefficient of lateral soil pressure; a – coefficient of soil constriction; $\boldsymbol{u}$ – vector of filtration speed; $\boldsymbol{D}$ – coefficient of convective diffusion; $\boldsymbol{D_T}$ – coefficient of thermal diffusion; C_m – concentration of soil saturation limit; γ_1 – coefficient of mass-transfer rate; σ – soil porosity; $\boldsymbol{\lambda}$ – coefficient of effective damp soil thermal conductivity; ρ – density of pore solution; c_{ρ} – specific heat of pore solution; c_T – soil volumetric heat; d – thickness of semi-permeable inclusion; k^{γ^*}, ν^{γ^*}, μ^{γ^*}, D^{γ^*}, $D_T^{\gamma^*}$, λ^{γ^*} – characteristics of semi-permeable inclusion γ^*; α – degree of ideality of semi-permeable inclusion, $0\le\alpha\le 1$; $\boldsymbol{n}$ – directed vector of directional cosine of the normal before semi-permeable inclusion; $h_0(x,y,z)$, $c_0(x,y,z)$, $T_0(x,y,z)$, H_1, H_2, c_1, c_2, T_1, T_2 – specified functions; $\Gamma_1\cup\Gamma_2\cup\Gamma_3\cup\Gamma_4\cup\Gamma_5\cup\Gamma_6\cup\Gamma_7\cup\Gamma_8=\Gamma$ – area border Ω.

Numerical solution of boundary value problem received with the help of finite element method (FEM). Considering H_0 – as space of vector-functions $s(x,y,z)=(s_1(x,y,z);s_2(x,y,z);s_3(x,y,z))$, each element $s_1(x,y,z),s_2(x,y,z),s_3(x,y,z)$ of which in area Ω (except limit points Γ and inclusions γ^*) belong to Sobolyev space $W_2^1(\Omega)$, besides that $s_1(x,y,z),s_2(x,y,z),s_3(x,y,z)$ obtain zero meaning in those parts of area Ω, where correspondingly for the next functions $h(x,y,z,t),c(x,y,z,t),T(x,y,z,t)$, limit conditions of the first kind are established.

Multiplying equations (1) – (3) and each of initial conditions (16) by functions $(s_1(x,y,z);s_2(x,y,z);s_3(x,y,z))\in H_0$ and having integrated received equations in area Ω, after use of the Gauss-Ostrogradsky formula, taking into account limit conditions (5) – (10) and matching conditions (11) – (14), we will receive

$$\iiint\limits_{\Omega}\frac{\partial h}{\partial t}s_1dxdydz+\frac{(1+e)(1+2\xi)}{3\gamma a}(\iiint\limits_{\Omega}\left(\boldsymbol{K}(c,T,e)\nabla h-\boldsymbol{\nu}(c)\nabla c-\boldsymbol{\mu}\nabla T\right)\times$$
$$\times\nabla s_1dxdydz+\frac{1}{d}\iint\limits_{\gamma^*}(k^{\gamma^*}[h]-\nu^{\gamma^*}[c]-\mu^{\gamma^*}[T])[s_1]d\Gamma)=0, \tag{17}$$

$$\iiint\limits_{\Omega}h(x,y,z,0)s_1(x,y,z)dxdydz=\iiint\limits_{\Omega}h_0(x,y,z)s_1(x,y,z)dxdydz, \tag{18}$$

$$\sigma\iiint\limits_{\Omega}\frac{\partial c}{\partial t}s_2dxdydz+\iiint\limits_{\Omega}(\boldsymbol{D}\nabla c,\nabla s_2)dxdydz+\iiint\limits_{\Omega}(\boldsymbol{D_T}\nabla T,\nabla s_2)dxdydz+$$
$$+\iiint\limits_{\Omega}(\boldsymbol{u},\ \nabla c)s_2dxdydz+\gamma_1\iiint\limits_{\Omega}cs_2dxdy+(1-\alpha)\times\iint\limits_{\gamma^*}((\frac{D^{\gamma^*}}{d}[c]+\frac{D_T^{\gamma^*}}{d}[T]-$$
$$-(\boldsymbol{u},\boldsymbol{n})_{\gamma^*}^{-}c^{-})[s_2]+(\boldsymbol{u},\boldsymbol{n})_{\gamma^*}^{-}[cs_2])d\Gamma=\gamma_1C_m\iiint\limits_{\Omega}s_2dxdydz, \tag{19}$$

$$\iiint\limits_{\Omega}c(x,y,z,0)s_2(x,y,z)dxdydz=\iiint\limits_{\Omega}c_0(x,y,z)s_2(x,y,z)dxdydz, \tag{20}$$

$$c_T\iiint\limits_{\Omega}\frac{\partial T}{\partial t}s_3dxdydz+\iiint\limits_{\Omega}\left(\lambda(c)\nabla T,\nabla s_3\right)dxdydz+\frac{1}{d}\iint\limits_{\gamma^*}((\frac{\lambda^{\gamma^*}}{d}[T]-\rho c_\rho\times$$
$$\times(\boldsymbol{u},\boldsymbol{n})_{\gamma^*}^{-}T^{-})[s_3]+\rho c_\rho(\boldsymbol{u},\boldsymbol{n})_{\gamma^*}^{-}[Ts_3])d\Gamma+\rho c_\rho\iiint\limits_{\Omega}(\boldsymbol{u},\nabla T)s_3dxdydz=0, \tag{21}$$

$$\iiint\limits_{\Omega}T(x,y,z,0)s_3(x,y,z)dxdydz=\iiint\limits_{\Omega}T_0(x,y,z)s_3(x,y,z)dxdydz. \tag{22}$$

We will seek approximate generalized solution in the following equation

$$\left(\hat{h}(x,y,z,t);\hat{c}(x,y,z,t);\hat{T}(x,y,z,t)\right)=(\sum_{i=1}^{n}a_i(t)N_i^{(1)}(x,y,z);\sum_{j=1}^{n}b_j(t)N_j^{(2)}(x,y,z);$$
$$\sum_{s=1}^{n}r_S(t)N_s^{(3)}(x,y,z)),\text{ де }\boldsymbol{N^i}=\left(N_i^{(1)};0;0\right),\boldsymbol{N^j}=\left(0;N_j^{(2)};0\right),\boldsymbol{N^k}=\left(0;0;N_k^{(3)}\right),$$

$i=\overline{1,n},\ \ j=\overline{1,n},\ \ k=\overline{1,n}$ – basic vector-functions of finite-dimensional subspace $M_0\subset H_0$, which are discontinuous on inclusion [5, 10, 11].

To solve the problem with the help of finite element method, we will discretise area Ω into tetrahedrons with double numeration of points placed on inclusion. Substituting form of approximate solution in equations (17) – (22), we will receive Cauchy problem for the system of non-linear differential equations relating to vector $\boldsymbol{U}(t)=(\boldsymbol{A}(t);\boldsymbol{B}(t);\boldsymbol{R}(t))$

$$\boldsymbol{M}^{(1)}\frac{d\boldsymbol{A}}{dt}+\boldsymbol{L}^{(1)}\boldsymbol{A}(t)=\boldsymbol{G}^{(1)}\boldsymbol{B}(t)+\boldsymbol{G}'^{(1)}\boldsymbol{R}(t)+\boldsymbol{F}^{(1)}, \tag{23}$$

$$\boldsymbol{M}^{(2)}\frac{d\boldsymbol{B}}{dt}+\boldsymbol{L}^{(2)}\boldsymbol{B}(t)=\boldsymbol{G}^{(2)}\boldsymbol{R}(t)+\boldsymbol{F}^{(2)}, \tag{24}$$

$$\boldsymbol{M}^{(3)}\frac{d\boldsymbol{R}}{dt}+\boldsymbol{L}^{(3)}\boldsymbol{R}(t)=\boldsymbol{F}^{(3)}, \tag{25}$$

$$\boldsymbol{M}^{(1)}\boldsymbol{A}^{(0)}=\tilde{\boldsymbol{F}}^{(1)},\ \tilde{\boldsymbol{M}}^{(2)}\boldsymbol{B}^{(0)}=\tilde{\boldsymbol{F}}^{(2)},\ \tilde{\boldsymbol{M}}^{(3)}\boldsymbol{R}^{(0)}=\tilde{\boldsymbol{F}}^{(3)}, \tag{26}$$

where

$$m_{ij}^{(1)}=\iiint_{\Omega}N_i^{(1)}N_j^{(1)}dxdydz,\quad f_i^{(1)}=0,\quad \tilde{f}_i^{(1)}=\iiint_{\Omega}\tilde{h}_0N_i^{(1)}dxdy,$$

$$l_{ij}^{(1)}=\frac{(1+e)(1+2\xi)}{3\gamma a}\iiint_{\Omega}(\boldsymbol{K}\nabla N_j^{(1)},\nabla N_i^{(1)})dxdydz+\frac{1}{d}\iint_{\gamma^*}k^{\gamma^*}[N_j^{(1)}][N_i^{(1)}]d\Gamma,$$

$$g_{ij}^{(1)}=\frac{(1+e)(1+2\xi)}{3\gamma a}\iiint_{\Omega}(\boldsymbol{\nu}\nabla N_j^{(2)},\nabla N_i^{(1)})dxdydz+\frac{1}{d}\iint_{\gamma^*}\nu^{\gamma^*}[N_j^{(2)}][N_i^{(1)}]d\Gamma,$$

$$g_{ij}'^{(1)}=\frac{(1+e)(1+2\xi)}{3\gamma a}\iiint_{\Omega}(\boldsymbol{\mu}\nabla N_j^{(3)},\nabla N_i^{(1)})dxdydz+\frac{1}{d}\iint_{\gamma^*}\mu^{\gamma^*}[N_j^{(3)}][N_i^{(1)}]d\Gamma,$$

$$m_{ij}^{(2)}=\sigma\iiint_{\Omega}N_i^{(2)}N_j^{(2)}dxdydz,\quad m_{ij}^{(3)}=c_T\iiint_{\Omega}N_i^{(3)}N_j^{(3)}dxdydz,$$

$$l_{ij}^{(2)}=\iiint_{\Omega}\left(\left(\boldsymbol{D}\nabla N_j^{(2)},\nabla N_i^{(2)}\right)+N_i^{(2)}\left(\boldsymbol{u},\nabla N_j^{(2)}\right)+\gamma_1N_i^{(2)}N_j^{(2)}\right)dxdydz+$$

$$+(1-\alpha)\iint_{\gamma^*}\left\{\left(\frac{D^{\gamma^*}}{d}\left[N_j^{(2)}\right]-(\boldsymbol{u},\boldsymbol{n})_{\gamma^*}^{-}N_j^{(2)-}\right)\left[N_i^{(2)}\right]+(\boldsymbol{u},\boldsymbol{n})_{\gamma^*}^{-}\left[N_j^{(2)}N_i^{(2)}\right]\right\}d\Gamma$$

$$g_{ij}^{(2)}=-\iiint_{\Omega}\left(\boldsymbol{D_T}\nabla N_j^{(3)},\nabla N_i^{(2)}\right)dxdydz-(1-\alpha)\iint_{\gamma^*}\frac{D_T^{\gamma^*}}{d}\left[N_j^{(3)}\right][N_i^{(2)}]d\Gamma,$$

$$f_i^{(2)}=\iiint_{\Omega}\gamma_1N_i^{(2)}C_mdxdydz,\ \tilde{m}_{ij}^{(2)}=\iiint_{\Omega}N_i^{(2)}N_j^{(2)}dxdydz,\ \tilde{f}_i^{(2)}=\iiint_{\Omega}c_0N_i^{(2)}dxdydz,$$

$$l_{ij}^{(3)}=\iiint_{\Omega}\left(\left(\boldsymbol{\lambda}\nabla N_j^{(3)},\nabla N_i^{(3)}\right)+\rho c_\rho N_i^{(3)}\left(\boldsymbol{u},\nabla N_j^{(3)}\right)\right)dxdydz+\frac{1}{d}\times$$

$$\times\iint_{\gamma^*}\left\{\left(\frac{\lambda^{\gamma^*}}{d}\left[N_j^{(3)}\right]-\rho c_\rho(\boldsymbol{u},\boldsymbol{n})_{\gamma^*}^{-}N_j^{(3)-}\right)\left[N_i^{(3)}\right]+\rho c_\rho(\boldsymbol{u},\boldsymbol{n})_{\gamma^*}^{-}[N_j^{(3)}N_i^{(3)}]\right\}d\Gamma,$$

$$f_i^{(3)}=0,\quad \tilde{m}_{ij}^{(3)}=\iiint_{\Omega}N_i^{(3)}N_j^{(3)}dxdydz,\quad \tilde{f}_i^{(3)}=\iiint_{\Omega}T_0N_i^{(3)}dxdydz.$$

We will receive approximate solution of the system of non-linear differential equations (23) – (25) with the help of the Crank-Nicolson scheme [9]

$$\boldsymbol{M}^{(3)}(\frac{\boldsymbol{R}^{(j+1)}-\boldsymbol{R}^{(j)}}{\tau})+\boldsymbol{L}^{(3)}(\boldsymbol{A}^{(j+1/2)},\boldsymbol{B}^{(j+1/2)},\boldsymbol{R}^{(j+1/2)})\boldsymbol{R}^{(j+1/2)}=\boldsymbol{F}^{(3)(j+1/2)},$$

$$\boldsymbol{M}^{(2)}(\frac{\boldsymbol{B}^{(j+1)}-\boldsymbol{B}^{(j)}}{\tau})+\boldsymbol{L}^{(2)}(\boldsymbol{A}^{(j+1/2)},\boldsymbol{B}^{(j+1/2)},\boldsymbol{R}^{(j+1/2)})\boldsymbol{B}^{(j+1/2)}=\boldsymbol{G}^{(2)}\boldsymbol{R}^{(j+1/2)}+\boldsymbol{F}^{(2)(j+1/2)},$$

$$\boldsymbol{M}^{(1)}(\frac{\boldsymbol{A}^{(j+1)}-\boldsymbol{A}^{(j)}}{\tau})+\boldsymbol{L}^{(1)}(\boldsymbol{B}^{(j+1/2)},\boldsymbol{R}^{(j+1/2)})\boldsymbol{A}^{(j+1/2)}=\boldsymbol{G}^{(1)}\boldsymbol{B}^{(j+1/2)}+\boldsymbol{F}^{(1)(j+1/2)},$$

where $\tau=\frac{t_0}{m_1}$, $j=0,1,2,...,m_1-1,$ $\boldsymbol{A}^{(j)}$ – value of elements at $t=\tau j$, $j=\overline{0,m_1-1};$ $\boldsymbol{A}^{(j+1/2)}=\frac{1}{2}(\boldsymbol{A}^{(j+1)}+\boldsymbol{A}^{(j)})$. Another vectors and matrices have similar notations.

Received scheme is non-linear relatively to required functions on time layers $(j+1), j=\overline{0,m_1-1}$. That is why, for its solution, it is necessary to use iterated methods. To avoid this, the scheme predicator-corrector can be used [15].

Results of numeral experiments. Three-dimensional problem of filtration consolidation of soil in the basis of hydro-technical construction taking into account the heat and salt transfer with the following basic data was investigated:

$e=0.7$, $\xi=0.75$, $a=5.12\cdot10^{-6}\,м^2/н$, $\gamma=10^4\,доба^{-1}$, $q=2\cdot10^5\,м$, $\nu_{11}=\nu_{22}=2.8\cdot10^{-5}\,м^5/кг\cdot доба$, $\mu_{11}=\mu_{22}=2.8\cdot10^{-6}\,м^2/град\cdot доба$, $D_{11}=D_{22}=0.02\,м^2/доба$, $k^{\gamma^*}=6\cdot10^{-7}\,м/доба$, $\nu^{\gamma^*}=10^{-5}\,м^5/кг\cdot доба$, $\mu^{\gamma^*}=10^{-6}\,м^2/град\cdot доба$, $d=0.2\,м$, $C_m=350\,г/літр$, $D^{\gamma^*}=0.0002\,м^2/доба$, $\alpha=0.1$, $\rho=1100\,кг/м^3$, $c_\rho=4.2\,кДж/кг\cdot град$, $c_T=2137\,кДж/м^3\cdot град$, $\gamma_1=0\,доба^{-1}$, $\lambda=108\,кДж/м\cdot град\cdot доба$, $\lambda^{\gamma^*}=108\,кДж/м\cdot град\cdot доба$, $H_1=8м$, $H_2=2м$, $c_1=350\,г/літр$, $T_1=30℃$, $c_0=5\,г/літр$, $T_1=20℃$, $\tau=10\,діб$.

(м-m, кг-kg, град-grade, доба-day, діб-days, літр-liter, кДж-kJ) Experimental dependencies for filtration coefficient and their approximation were taken from work [5].

Soil area is right-angled parallelepiped $10м\times10м\times10м$. Area of load application q is located between right lines $x=2.9м$ і $x=7.1м$ in plane $z=0$.

Semi-permeable inclusion is located in the depth of 5 m. It is in the form of rectangle with tops $(3;3)$, $(3;7)$, $(8;7)$, $(8;3)$.

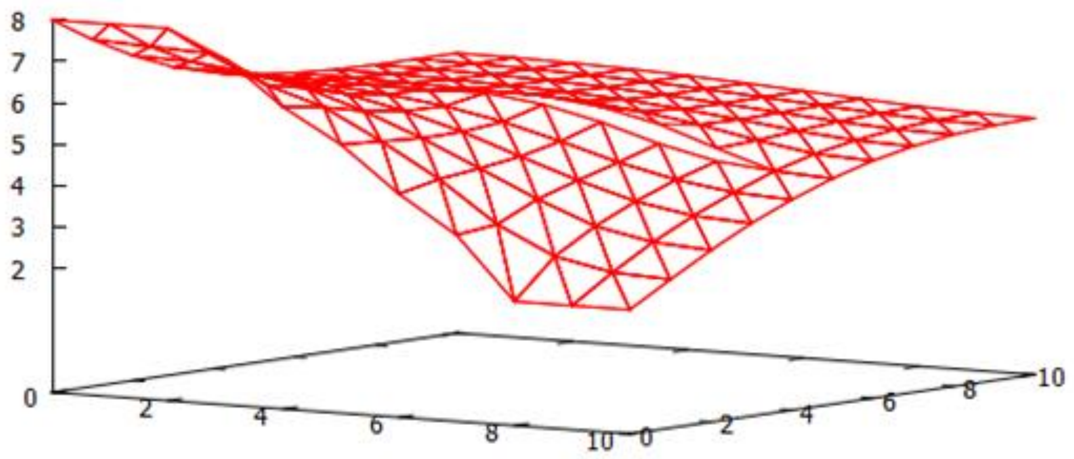

Figure 2. Distribution of the pressures when t=720 days with taking into account heat and salt transfer

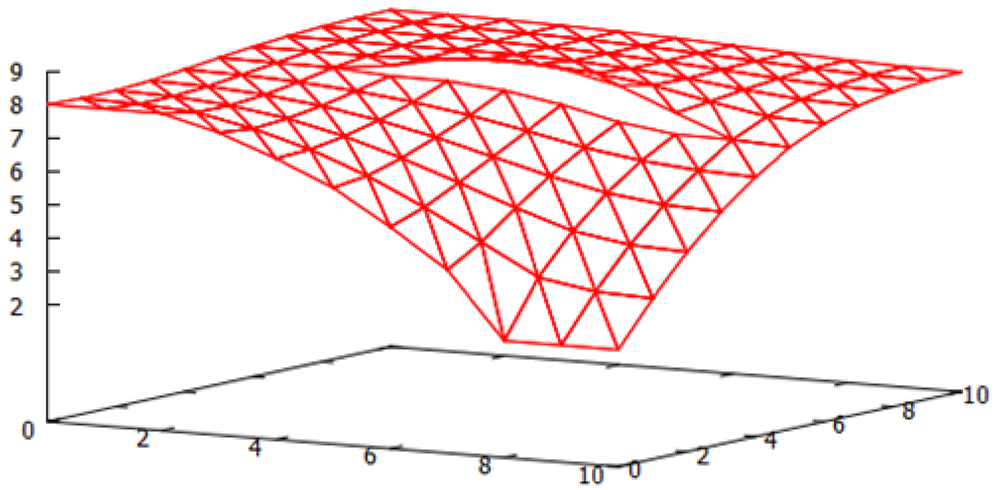

Figure 3. Distribution of the pressures when t=720 days without taking into account heat and salt transfer

Table № 1.
Jumps of pressure on inclusion taking into account the heat and salt transfer (dimension is defined by meters)

x	y	z	t=30 days	t=180 days	t=360 days	t=720 days	t=1080 days	t=1440 days
3	5	5	0,2152	-0,6366	-0,3934	0,0359	0,2196	0,2920
4,643	5,893	5	0,2275	-1,3838	-0,9825	-0,1550	0,2071	0,3487
4,000	5,000	5	0,3130	-1,2567	-0,8491	-0,0513	0,2932	0,4273
7,274	6,310	5	-0,1634	-1,2844	-0,9507	-0,3467	-0,0744	0,0359
5,714	6,786	5	0,0852	-1,2145	-0,8825	-0,2317	0,0565	0,1698
5	5	5	0,2893	-1,6253	-1,1573	-0,2084	0,2063	0,3681
7,786	5,714	5	-0,2860	-1,2546	-0,9377	-0,3673	-0,1076	-0,0007
8,000	5,000	5	-0,2702	-1,0523	-0,7867	-0,3129	-0,0971	-0,0079
4,286	3,214	5	0,2678	-0,8096	-0,5465	-0,0290	0,1964	0,2830
4,643	4,107	5	0,2960	-1,3766	-0,9612	-0,1286	0,2350	0,3765
6,381	4,048	5	-0,0007	-1,5514	-1,1363	-0,3494	0,0009	0,1407
7,238	3,571	5	-0,1883	-1,0700	-0,7912	-0,2907	-0,0637	0,0289

Table № 2.
Jumps of pressure on inclusion without taking into account the heat and salt transfer
(dimension isdefined by meters)

x	y	z	t=30 days	t=180 days	t=360 days	t=720 days	t=1440 days
3	5	5	0,2316	-0,0170	-0,2329	-0,3005	-0,2203
4,643	5,893	5	0,2456	-0,1672	-0,6451	-0,8011	-0,6495
4,000	5,000	5	0,3359	-0,0941	-0,5349	-0,6750	-0,5284
7,274	6,310	5	-0,1379	-0,4028	-0,6964	-0,7901	-0,6763
5,714	6,786	5	0,1046	-0,2713	-0,6289	-0,7440	-0,6219
5	5	5	0,3098	-0,2185	-0,7534	-0,9289	-0,7572
7,786	5,714	5	-0,2554	-0,4237	-0,6903	-0,7741	-0,6655
8,000	5,000	5	-0,2433	-0,3595	-0,5777	-0,6461	-0,5561
4,286	3,214	5	0,2872	-0,0784	-0,3657	-0,4557	-0,3576
4,643	4,107	5	0,3181	-0,1568	-0,6284	-0,7822	-0,6291
6,381	4,048	5	0,0247	-0,3933	-0,8021	-0,9342	-0,7886
7,238	3,571	5	-0,1595	-0,3425	-0,5852	-0,6608	-0,5648

Considering the stated above results (Fig. 2, 3), we can see how the heat and salt transfer influence the distribution of the pressures in the soil. Taking into account the given processes, filtration consolidation takes place nearly in one and a half times more rapid than while without taking into account the heat and salt transfer.

As we can see in Table 1, jump of the pressures is observed on semi-permeable inclusion, besides almost at all points it is negative and then it becomes positive, (taking into account the heat and salt transfer at time layer t = 1080 days). Without considering the heat and salt transfer, the change of jump of the pressure takes place more slowly without change of a sign.

Conclusions. The model of filtration consolidation of soils with semi-permeable inclusions in three-dimensional case has been constructed. The influence of the heat and salt transfer, taking into account the level of ideality of semi-permeable inclusion, on the distributions of pressures has been shown. Jump of the pressure on the inclusion was found and the dynamics of its change was investigated. A number of numerical experiments has been conducted.

Theoretical investigations of quality characteristics of obtained numerical solutions are planned to be done in future.

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УДК 627.324.2/3:532.72

МЕТОД СКІНЧЕННИХ ЕЛЕМЕНТІВ У ПРОСТОРОВІЙ ЗАДАЧІ ФІЛЬТРАЦІЙНОЇ КОНСОЛІДАЦІЇ ҐРУНТІВ З ТОНКИМИ НАПІВПРОНИКНИМИ ВКЛЮЧЕННЯМИ В УМОВАХ ВПЛИВУ ТЕПЛОСОЛЕПЕРЕНЕСЕННЯ

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Резюме. *Сформовано математичну модель фільтраційної консолідації ґрунтів з урахуванням наявних там тонких напівпроникних включень у просторовому випадку. Для чисельного розв'язання тривимірної задачі використано метод скінченних елементів. Проведено ряд чисельних експериментів та показано вплив теплосолеперенесення і напівпроникних включень на розподіл напорів у ґрунтових пластах.*

Ключові слова*: фільтраційна консолідація, умови спряження, напівпроникне включення, метод скінченних елементів.*

Отримано 11.01.16