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DESTRUCTION ZONE NEAR THE TIP OF INTERFACIAL CRACK AT A PREVAILING TENSILE LOADING

Mykhailo Dudyk

Pavlo Tychyna Uman State Pedagogical University, Uman, Ukraine

Summary. *Under the plane strain conditions at the prevailing tensile loading by Wiener-Hopf method the solutions of problems about the calculation of a small-scale destruction zone in the pre-fracture zone part, which is adjacent to the interface crack tip, located on a flat interface of two different materials, taking into account and ignoring the contact of the lips near the tip have been found. The influence of the elastic characteristics of joining materials and loading configurations on the parameters of the destruction zone has been investigated.*

Key words: *interfacial crack, pre-fracture zone, destruction zone, contact of the lips.*

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Problem setting. Experimental investigations of fracture processes near the crack tip revealed existence in their vicinity of pre-fracture zones with a complex structure and include in the part adjacent directly to the tip of the relatively small area of material destruction with a very high deformation level [1, 2]. The complex model of pre-fracture zone at the end of interfacial crack [3, 4] except a destruction zone takes into account the contact of the lips. In [5] within the framework of complex model of pre-fracture zone the calculation of a small-scale contact zone has been done in the presence of a more developed pre-fracture zone, but the sizes of the destruction zone has not been established. Under the prevailing tensile loadings in the direction perpendicular to the plane of interfacial crack the size of the area of the lips contact may be much smaller than the sizes of the pre-fracture zone and the destruction zone as well.

The aim of the work is to find the parameters of destruction zone taking into account the contact of the lips and ignoring it.

1. Computation of a destruction zone taking into account the contact of the lips. Statement of the problem. Under the plane strain conditions, we consider the problem computing the destruction zone in the lateral pre-fracture zone propagating from the tip of the crack, located in a piecewise homogeneous body on a straight-line interface of two different isotropic elastic material with Young's moduli E_1, E_2 and Poisson's ratios ν_1, ν_2, taking into account the contact of the lips near the crack tip. Due to the tearing nature of prefracture zone and in accordance with the localization hypothesis it can be simulated as inclined at an angle α to the interface with a straight line of rupture of a normal displacement of the length l propagating from the crack tip into the first material, which is assumed to be less crack-resistant. According to Leonov-Panasiuk's model normal stress on the line of rupture is equal to the resistance of the first material to separation σ_1 [6]. The area of destruction of the material in the pre-tip of the pre-fracture zone, which is characterized by a high level of both normal and shear deformations, will be simulated as a rupture line of length d, where both the normal and tangential displacement are undergone the rupture, and a tangential stress is equal to the shear resistance of the first material τ_{1s} [4]. We assume that the length of the zone of destruction is much less than the length of the contact zone s, which in its turn is significantly less than the length of the entire pre-fracture zone ($d = s = l$). It allows us to regard the studied body as a piecewise homogeneous plane, containing at the interface a semi-infinite zone of the contact sliding of the lips, interacting according to the law of dry friction, from the O tip at an angle α to the interface a semi-infinite straight line of rupture propagates which consists of two sections

(Fig.1). In the section OO', which is adjacent to the crack tip, both normal and shear displacement experience the rupture, and normal and tangential stresses are equal to σ_1 i τ_{1s} . In the second section only the the normal displacement is ruptured and the normal stress is equal to σ_1.

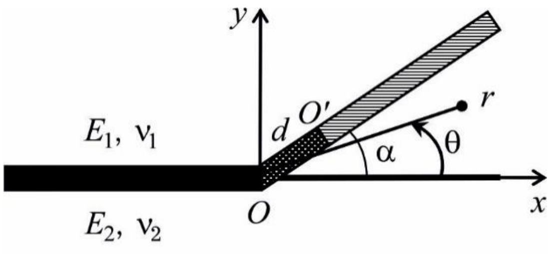


Figure 1. The computational scheme of the problem

This model corresponds to the static boundary problem of elasticity theory with boundary conditions

$$\theta = 0: \ <\sigma_\theta>=<\tau_{r\theta}>=0, \quad <u_\theta>=<u_r>=0;$$
$$\theta = \pm\pi: \ <\sigma_\theta>=<\tau_{r\theta}>=0, \quad <u_\theta>=0, \ \ \tau_{r\theta} = -\mu\sigma_\theta;$$
$$\theta = \alpha: \quad <\sigma_\theta>=<\tau_{r\theta}>=0; \ \ \theta = \alpha, \ \ \sigma_\theta = \sigma_1;$$

$$\theta = \alpha, \ \ r < d: \ \ \tau_{r\theta} = \tau_1 = \pm\tau_{1s}; \ \ \theta = \alpha, \ \ r > d: \quad <u_r>=0, \tag{1}$$

where $\langle f \rangle$ is the jump of the quantity f , μ is coefficient of friction, sign τ_1 is determined by the sign of tangential stress in the pre-fracture zone part adjacented to the crack tip.

At infinity the principal terms in the stresses expansions into asymptotic series coincide with the principal terms in the stresses expansions asymptotic series near the crack tip in the problem, which is similar to this, in case $d = 0$ (without destruction area) and with finite area of the contact of the lips, the solution of which has been found [5]. In particular, from [5] we can find:

$$\theta = \alpha, \ \ r \to \infty: \ \ \tau_{r\theta} = 2(1+\kappa_1)\sigma_1 \left[\tilde{C}_0 + \sum_k \tilde{C}_k r^{\lambda'_k} \right] + o\left(1/r\right), \tag{2}$$

$$\tilde{C}_0 = -\frac{C_0 S'(-1)}{\pi D(-1)}, \ \tilde{C}_k = \frac{S(-1-\lambda'_k)Q^+(-1-\lambda'_k)G_1^+(-1-\lambda'_k)}{\lambda'_k \sin\lambda'_k \pi D'(-1-\lambda'_k)} s^{-\lambda'_k} \times$$
$$\times \sum_i \frac{C_i s^{\lambda_i - \lambda'_k}}{Q^+(-1-\lambda_i)G_1^+(-1-\lambda_i)} \frac{\lambda_i}{(\lambda_i - \lambda'_k)}, \ \ S(p) = S_0(p) + \mu S_1(p),$$
$$S_0(p) = [(1+\kappa_1)^2 - 4(1+\kappa_1)(1-e)\sin^2 p\alpha - 4(1+e\kappa_2)(1-e)t_1(p)]s_{01}(p) +$$
$$+e(1+\kappa_2)[(1+\kappa_1)s_{02}(p) - e(1+\kappa_2)s_{03}(p) - 2(1-e)t_2(p)t_3(p)],$$
$$s_{01}(p) = (p+1)\sin\alpha \sin p(\pi-\alpha)\sin p\pi,$$
$$s_{02}(p) = 2(p+1)\sin\alpha \cos p(\pi+\alpha)\sin p\alpha \sin p(\pi-\alpha) - t_2(p)\sin(p+1)\alpha,$$
$$s_{03}(p) = 2p^3 \sin^3\alpha \cos p\alpha - p^2 \sin^2\alpha \sin(p-1)\alpha - p\sin\alpha \sin p\pi \sin p(\pi-\alpha) +$$
$$+\sin p\alpha \sin p(\pi-\alpha)\sin[p(\pi-\alpha)-\alpha];$$

$$S_1(p)=4(1+e\kappa_2)(1-e)\sin p\pi\, t_1(p)t_4(p)+e(1+\kappa_2)t_2(p)[(1+\kappa_1)\cos(p+1)\alpha+$$
$$+2(1+e\kappa_2)(p+1)\sin\alpha\sin p\alpha]-(1+\kappa_1)[2e(1+\kappa_2)\sin p\alpha\cos p(\pi+\alpha)-$$
$$-4(1-e)\sin p\pi\sin^2 p\alpha+(1+\kappa_1)\sin p\pi]t_4(p)+e^2(1+\kappa_2)^2 s_{11}(p),$$
$$s_{11}(p)=p^2\sin^2\alpha\cos(p+1)\alpha+p\sin\alpha\cos p(\pi-\alpha)\sin p(\pi-2\alpha)-$$
$$-\sin p\alpha\sin p(\pi-\alpha)\cos[p(\pi-\alpha)+\alpha];$$
$$t_1(p)=p^2\sin^2\alpha-\sin^2 p\alpha,\; t_2(p)=p^2\sin^2\alpha-\sin^2 p(\pi-\alpha),$$
$$t_3(p)=p\sin\alpha\cos p\alpha-\cos\alpha\sin p\alpha,\;\; t_4(p)=p\sin\alpha\cos p(\pi-\alpha)+\cos\alpha\sin p(\pi-\alpha),$$
$$S'(p)=\partial S(p)/\partial p,\;\; D'(p)=\partial D(p)/\partial p,\;\; \kappa_{1(2)}=3-4\nu_{1(2)},\;\; e=\frac{E_1}{E_2}\frac{1+\nu_2}{1+\nu_1};$$

a constant C_0, degrees $\lambda_i>-1$ i $\lambda_k'>-1$ in expansions of asymptotic fields of stresses into series along the distance to the tip taking into consideration only the zone of pre-fracture or the zone of pre-fracture and small-scale area of lips contact, accordingly, the functions $D(p)$, $Q^+(p)$, $G_1^+(p)$ and the length of the contact zone are defined in [5]. Asymptotically the largest contribution to the tangential stress in the pre-fracture zone near the crack tip has been done by the term, which corresponds to the smallest in the interval (–1, 0) degree λ_1' in expansion (2), which is a stress singularity index in the area r << s. The sign of this term determines the sign τ_1 in the destruction zone: accordingly (2) $\tau_1=\tau_{1s}\,\mathrm{sgn}(\tilde{C}_1)$.

Solution of the problem and numerical analysis of the results. The boundary problem of elasticity has been formulated (1) – (2) which is similar to the boundary problem about computing of the destruction zone at the end of interfacial crack with a significant contact of the lips and a small-scale lateral pre-fracture zone that has been solved in [7], being different from it by the condition at infinity (2). Using the obtained solution in [7], taking into account the differences in conditions at the infinity we came to the transcendental equation for the calculation the length of the destruction zone:

$$\sum_k \frac{\tilde{C}_k d^{\lambda_k'} K^+(-1-\lambda_k')J_1(0)}{(1+\lambda_k')K^+(-1)J_1(\lambda_k')}=\frac{\tau_1}{2(1+\kappa_1)\sigma_1}-\tilde{C}_0, \tag{3}$$

$$K^+(p)=\frac{\Gamma(1-p)}{\Gamma(1/2-p)},\;\; J_1(x)=\exp\left[\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\ln H(it)}{1+x+it}dt\right],\;\; H(p)=\frac{\cos p\pi D_2(p)}{2\sin^2 p\pi D_1(p)};$$

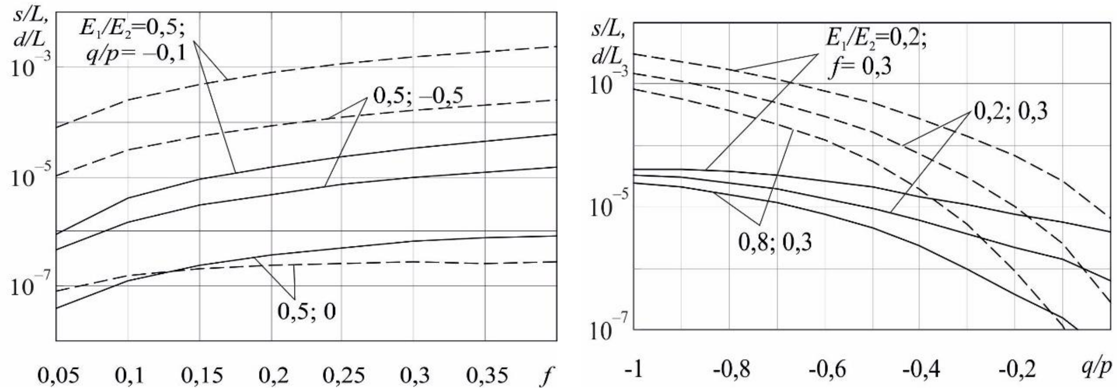
$\Gamma(p)$ is the gamma function; functions $D_1(p)$ i $D_2(p)$ are defined in [7].

The emergence of the destruction zone changes the stress-strain state near the crack tip which at distances $r=\; d$ will be characterized by the stresses singularity index λ_{d1}, which is defined as the smallest one from the interval (–1, 0) the root of the equation $D_2(-1-x)=0$. In addition, the destruction of the material leads to the nonzero shear displacement of the lips at the tip of the crack which is equal, accordingly to [7], to:

$$\delta=-\frac{4(1-\nu_1^2)\sigma_1}{E_1}\frac{2(1+\kappa_1)d}{\sqrt{\pi H(0)}\cos\alpha}\sum_k\frac{\tilde{C}_k d^{\lambda_k'}K^+(-1-\lambda_k')}{J_1(\lambda_k')}\frac{\lambda_k'}{(1+\lambda_k')^2}. \tag{4}$$

To analyze the obtained solution, we should consider a piecewise homogeneous plane with interfacial crack length L, which is loaded at infinity with the tensile normal stress $\sigma_y = p > 0$ and tangential stress $\tau_{xy} = q$. The parameters of a pre-fracture zone and the contact area are determined in accordance with [5, 8, 9]. In every calculation the coefficient of friction is $\mu = -0.5$ and in both parts of the article is $\nu_1 = \nu_2 = 0.3$.

In Fig. 2 the dependence of the length of destruction zone (solid lines) and the sizes of the area of the contact of the lips (dashed lines) on the loading for comparison are showed. The length of the destruction zone increases with the increase of dimensionless loading module $f = \sqrt{p^2 + q^2} / \sigma_1$ (fig. 2*a*), but it decreases with increasing the ratio of normal and tangential stresses *q/p*, which defines the configuration of loading (fig. 2*b*).



a) *b*)

Figure 2. The dependence of the lengths of the destruction zone *d* (solid lines) and the contact area of the lips *s* (dashed lines) on the loading module *f* (*a*) and on the loading configuration *q/p* (*b*).

The analysis of the graphs shows that by ratio of normal and tangential stresses within $-1 \le q/p \le -0{,}5$ the destruction zone is much smaller than the size of the contact area, which ensures the implementation of the initial condition $d = s$ of the model, whereas under decreasing the contribution in the loading of tangent component q, this condition is violated.

Fig. 3 shows the results of calculations of stresses singularity at different distances to the tip: λ_1 if $s = r = l$, λ_1' if $d = r = s$ and λ_{d1} if $r = d$. As singularity indexes satisfy inequalities $\lambda_1' < \lambda_1 < \lambda_{d1}$, we can make a conclusion that at distances $s = r = l$ the level of stresses concentration is below the square root, but due to the formation of small-scale contact zone it greatly increases at distances $d = r = s$, and the establishment of a destruction zone eliminates the strengthening of stresses concentration at distances $r = d$.

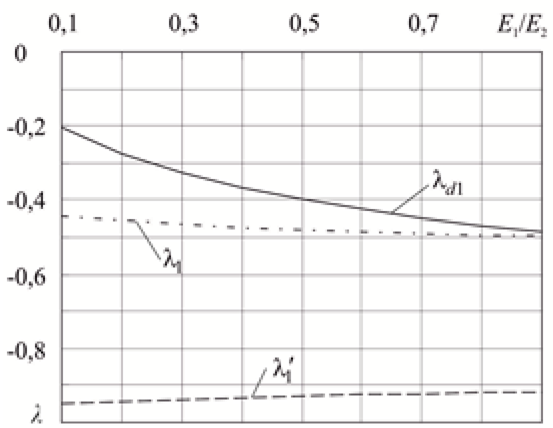


Figure 3. The dependence of the singularity indexes of stresses at different distances to the crack tip on the Young's moduli ratio E_1/E_2 of joining materials for $f = 0{,}3$, $q/p = -0{,}5$.

Taking into account the lateral pre-fracture zone and the contact of the lips the opening of the crack at its tip will be zero, so that according to deformation criterion the crack start is impossible [4]. However, the appearance of destruction zones causes the relative shear of crack lips in its tip, resulting in a shear crack opening δ (4). Fig. 4 shows the dependence of the normalized shear opening $\delta' = \delta E_1 / 4L(1-\nu_1^{\,2})\sigma_1$ on the magnitude of the external loading at some of its configuration and some values of Young's modulus ratio of joining materials.

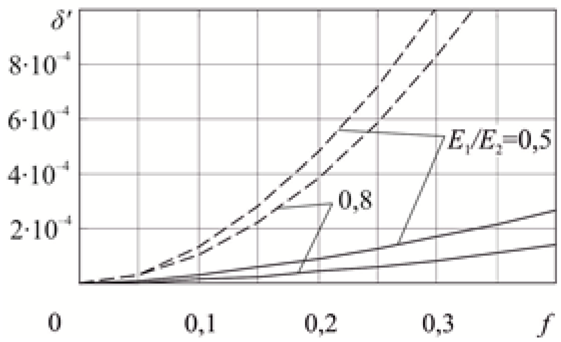


Figure 4. The normalized shear opening of the crack as function of the loading *f* for *q/p* = – 0,5 (solid lines) and *q/p* = – 1 (dashed lines).

According to the calculations (Fig. 4), the crack opening increases with the magnitude of the external loading. It behaves like the size of the contact area and the length of the zone of destruction (Fig. 3): it decreases due to the increasing of normal tensile stress contribution and due to the approach of the elastic characteristics of joining materials ($E_1 \to E_2$).

2. Computation of the destruction zone ignoring the contact of the lips.

Formulation of the problem and its solution. As it has been established above, with the significant prevalence in the external loading of tensile efforts in a perpendicular direction to the crack plane, the length of the contact zone may be smaller then the length of the zone of destruction and the solution that has been received in the previous part of the article is incorrect. In this regard, in this part the problem about the destruction zone in the lateral small-scale pre-fracture zone ignoring the contact of the lips is solved. Assuming the lips of the crack are free from loading we come to the boundary problem of elasticity which is similar to that one discussed above with the replacement of conditions in (1) on the lips of the crack with the conditions

$$\theta = \pm\pi: \quad \sigma_\theta = \tau_{r\theta} = 0,$$

and we formulate the condition at infinity due to the requirement the possibility of sewing together the wanted solution with the asymptotic solution if $r \to 0$ in the problem about the lateral small-scale pre-fracture zone near the tip of the open interfacial crack [8, 9]:

$$\theta = \alpha, \quad r \to \infty, \quad \tau_{r\theta} = \sigma_1\left[C_0 + \sum_i C_i r^{\lambda_i}\right] + o\left(\frac{1}{r}\right),$$

$$C_0 = \frac{-2u'(-1)}{D_1'(-1)}, \quad u'(p) = du(p)/dp, \quad D_1'(p) = \partial D_1(p)/\partial p,$$

$$C_i = \frac{4(1+\lambda_i)u(-1-\lambda_i)G^+(-1-\lambda_i)}{\sigma_1\lambda_i D_1'(-1-\lambda_i)K^+(-1-\lambda_i)} l^{-\lambda_i}\,\mathrm{Re}\left[\frac{F(\alpha)l^{-0,5+i\omega}K^+(-0,5-i\omega)}{(0,5+i\omega)G^+(-0,5-i\omega)}\frac{1-2\omega}{1+2\lambda_k-2\omega}\right],$$

$$u(p) = -(1+e\kappa_2)^2 u_1(p) + (1-e)(1+e\kappa_2)u_2(p) + (e+\kappa_1)(1+e\kappa_2)u_3(p) +$$
$$+(1-e)(e+\kappa_1)u_4(p) + (e+\kappa_1)^2 \sin p\pi u_4(p),$$

$u_1(p) = p(p+1)\sin^2\alpha \sin p(\pi - 2\alpha),$

$u_2(p) = p^2 \sin^2\alpha \sin p(3\pi - 2\alpha) + 2p\sin^2\alpha[\cos 2p\pi \sin p(\pi - 2\alpha) +$

$+\sin p\alpha \cos p(\pi + \alpha)] - \sin p\pi \sin^2 p(\pi - \alpha),$

$u_3(p) = p^2 \sin^2\alpha \sin p(\pi + 2\alpha) + 2p\sin^2\alpha \sin p\alpha \cos p(\pi + \alpha)] - \sin p\pi \sin^2 p(\pi - \alpha),$

$u_4(p) = p^2 \sin^2\alpha \sin p(\pi - 2\alpha) + 2p\sin^2\alpha \sin p(\pi - \alpha)\cos p\alpha + \sin p\pi \sin^2 p(\pi - \alpha),$

$u_5(p) = p\sin^2\alpha + \sin^2 p(\pi - \alpha);$

functions $F(\alpha)$, $D_1(p)$ i $G^+(p)$, the length l of the pre-fracture zone and the angle of its inclination α are defined in [8, 9]; λ_i are the roots of the equation $D_1(-1-x)=0$, that satisfy the condition $\operatorname{Re}\lambda_i > -1$.

The solution of the formulated problem was received by means of the Wiener-Hopf method which is similar to the solution of the analogous problem in [7] and leads to the equation for the determination the length of the zone of destruction *d* and to the expression for crack opening δ in its tip:

$$\sum_i \frac{C_i d^{\lambda_i} K^+\left(-1-\lambda_i\right) J_2\left(0\right)}{(1+\lambda_i)K^+\left(-1\right) J_2\left(\lambda_i\right)} = \frac{\tau_1}{\sigma_1} - C_0, \tag{5}$$

$$J_2(x) = \exp\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{\ln G_1\left(it\right)}{1+x+it}dt\right], \quad G_1\left(p\right) = \frac{4t_1(p)D_3\left(p\right)}{\sin p\pi D_1\left(p\right)};$$

$D_3(p) = (1+\kappa_1)^2 \sin^2 p\pi - D_{31}(p)t_2(p) + \sin p\alpha[D_{32}(p)\cos p\alpha - D_{33}(p)\sin p\alpha],$

$D_{31}(p) = e^2(1+\kappa_2)^2 + 4(1-e)(1+e\kappa_2)\sin^2 p\pi, \quad D_{32}(p) = (1+\kappa_1)e(1+\kappa_2)\sin 2p\pi,$

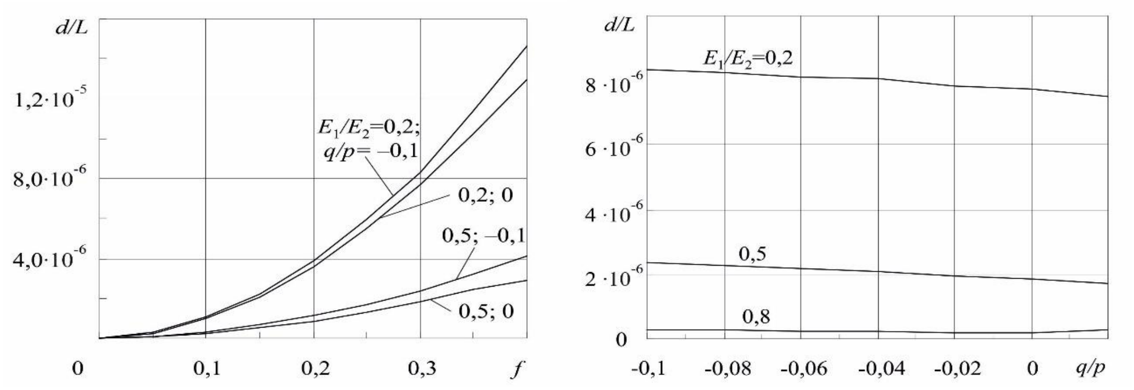
$D_{33}(p) = 2(1+\kappa_1)[e(1+\kappa_2) + 2(1-e)]\sin^2 p\pi;$

$$\delta = -\frac{2(1-{\nu_1}^2)}{E_1}\sigma_1 d\sqrt{\pi G_1(0)}\left[\frac{(\pi-\alpha)\sin\alpha}{(\pi-\alpha)^2 - \sin^2\alpha} + \frac{e(1+\kappa_2)\delta_1}{\delta_2}\right]\sum_i \frac{C_i d^{\lambda_i}\lambda_i K^+(-1-\lambda_i)}{(1+\lambda_i)^2 J_2(\lambda_i)}, \tag{6}$$

$\delta_1 = [e(1+\kappa_2)\alpha + (1+\kappa_1)\pi]\sin\alpha,$

$\delta_2 = (1+\kappa_1)^2\pi^2 + 2(1+\kappa_1)e(1+\kappa_2)\pi\alpha + e^2(1+\kappa_2)^2(\alpha^2 - \sin^2\alpha).$

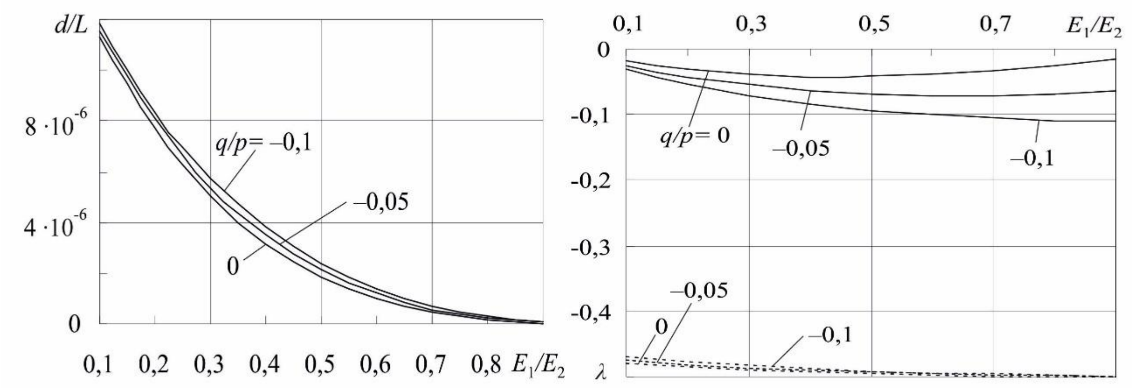
Analysis of numerical calculations. The numerical analysis of the obtained solution was made for the same body configuration and loading, as in the previous part of the article, but considering the ratio of tangential and normal stresses $|q|/p \le 0{,}1$, for which the ignoring of the contact of the lips is possible. The length of the destruction zone increases with the increasing of the loading module (Fig. 5*a*) and decreases with the increasing ratio of tangent and normal stresses (Fig. 5*b*). Moreover, its numerical values are of the same order of the length of the destruction zone which were found with the same parameters of materials and of loading in the presence of a contact zone in the previous part of the article (Fig. 2*b*).



a) *b*)

Figure 5. The dependence of the length of the destruction zone on the loading module (*a*) and on the loading configuration for f = 0,3 (*b*).

When approaching the elastic characteristics of joining materials, the length of the destruction zone tends to 0 (Fig. 6*a*), as well as according to the presence of the contact of the lips (Fig. 2*b*). This could mean more likely the development of the destruction zone in the other direction which is different from the orientation of pre-fracture zone. Fig. 6*b* shows that the appearance of the destruction zone reduces the level of stresses singularity at the crack tip almost from the square root which is caused by the pre-fracture zone (λ_1, dashed lines), to a significantly lower level of order $\lambda_{d1} \geq -0{,}1$ (solid lines).



a) *b*)

Figure 6. The dependence of the length of the destruction zone (*a*) and of the singularity index (*b*) on Young's moduli ratio E_1/E_2 of joining materials for f = 0,3.

The crack opening (6) in its tip within the framework of the investigated model turns out to be negative, indicating a possible contact of the lips, but the expected size of the contact area is assumed to be so small that defeats the purpose of its calculation.

Conclusions. Under the plane strain conditions at the prevailing tensile loading by Wiener-Hopf method the solutions of problems about the calculation of a small scale destruction zone in the lateral pre-fracture zone part adjacent to the interface crack tip taking into account the lips contact near the tip and ignoring it have been found. The equation for the calculations of a destruction zone length and the singularity indexes of the stresses at the distances to the crack tip which are much less then the sizes of a destruction zone length has been obtained. The expressions for the calculations of the crack opening have been deduced. On the basis of the numerical calculations the dependence of zone parameters on the configuration of external loading and the elastic characteristics of joining materials have been studied. The increase of zone length at the increase of loading module and its decrease at the increase of ratio of normal and tangential components of loading have been discovered. The weakening of stresses singularity near the crack tip after the formation of destruction zone has been displayed.

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ЗОНА ДЕСТРУКЦІЇ БІЛЯ ВЕРШИНИ МІЖФАЗНОЇ ТРІЩИНИ ПРИ ПЕРЕВАЖАЮЧИХ РОЗТЯГУВАЛЬНИХ НАВАНТАЖЕННЯХ

Михайло Дудик

Уманський державний педагогічний університет імені Павла Тичини, Умань, Україна

Резюме. *В умовах плоскої деформації при переважаючих розтягувальних навантаженнях за допомогою методу Вінера-Гопфа виконано розрахунок маломасштабної зони деструкції у частині зони передруйнування, прилеглій до вершини тріщини, розташованої на плоскій межі поділу двох різних матеріалів, за наявності і відсутності контакту берегів біля вершини. Досліджено вплив на параметри зони деструкції пружних характеристик з'єднаних матеріалів і конфігурації навантаження.*

Ключові слова*: міжфазна тріщина, зона передруйнування, зона деструкції, контакт берегів.*

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