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STRESS DISTRIBUTION IN AN INFINITE ORTHOTROPIC PLATE WITH PARTLY REINFORCED ELLIPTICAL CONTOUR

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Resume. *The work presents approximate solution of the problem of partial reinforcement of elliptical aperture contour in an infinite orthotropic plate with elastic edge whose bonding surface with the plate does not coincide with its axial surface. Simulating reinforcing beam with a curved rod having constant rectangular cross-section, we built a system of integral equations to determine the contact forces between the plate and the reinforcement, and functions to determine internal forces in the reinforcement. Numerical implementation of the task has been carried out by mechanical quadrature and collocation.*

Key words: *orthotropic plate, reinforcing edge, contact efforts, singular integral equations*.

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Problem setting. Intensive development of modern technology and construction calls for extensive use of plates with holes made of composite materials. To reduce the high stress concentration around the holes in the plates their contours are reinforced with open-ended elastic beams. Study of stressed state of the plate in the vicinity of reinforcement sections is one of the urgent problems of mechanics of contact interaction of massive and thin elastic bodies.

Tasks on partial reinforcement of contours of curved holes in isotropic or orthotropic plates that are in conditions of generalized flat stress, are thoroughly investigated for cases where reinforcement is modeled with an elastic line of constant or variable stiffness in tension (compression) and bend [1 – 3]. It is believed that the line of reinforcement and plate junction coincides with its geometrical axis.

Analysis of recent research and published works. Research [4] offers the solution to the problem of partial reinforcement of circle-shaped contour in an infinite isotropic plate and elastic disc with rods of rectangular section through which concentrated force load is passed to the plate. For curved openings in isotropic and orthotropic plates such problems were not considered.

This paper offers a numerical and analytical solution of the problem of partial reinforcement of contour of elliptical opening in the infinite orthotropic plate with an elastic curved rod.

The aim is to determine the contact tensions in the line of connection of the plate with reinforcing beam and to research the impact of plate material orthotropy and physical and geometrical parameters of the beam on its stress state.

Formulation of the problem. Consider an infinite orthotropic plate with thickness $2h$ with an elliptical opening limited by smooth cylindrical surface. Let us denote the line of surface intersection with the middle plate area by Γ and call it contour of the hole.

The system of Cartesian (x, y) and polar (r, δ) coordinates with pole at the center of the opening is set so that axis Ox coincides with the polar axis and ellipse symmetry axis and determines one of the main areas of plate material orthotropy. We believe that the plate is in state of generalized flat stress generated by forces p and q evenly distributed to infinity and acting in the middle plane of the plate in the direction of the coordinate axes (Figure 1).

Suppose that at the section $\Gamma_1 \equiv \left[\alpha_0^*, \beta_0^*\right]$ where α_0^*, β_0^* are polar angles, and contour Γ is reinforced with thin elastic beam of constant cross-section in the shape of a rectangle with width 2η and height $2h_0$.

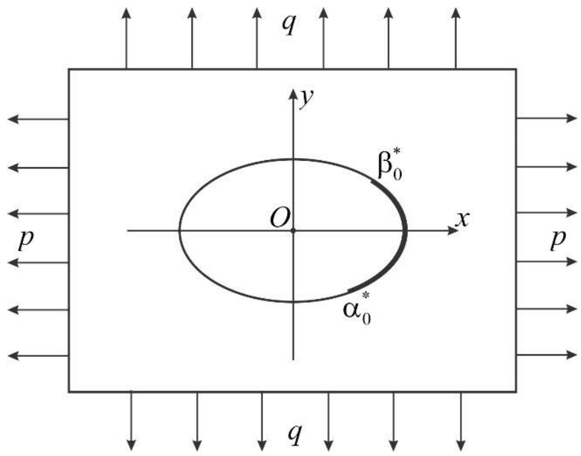

Figure 1. Loading diagram of the plate

The form of elliptical hole in an infinite plate is defined by function [5]

$$z = \omega(\zeta) = R^* \left(\zeta + \frac{\varepsilon}{\zeta} \right), \qquad (1)$$

performing conformal mapping of the exterior of the unit circle γ in the plane of the area $\zeta = \tilde{\rho} e^{i\lambda}$ occupied by the middle plane of the plate. Here $R^* = \frac{a+b}{2} = 1$ is the characteristic size of the hole; $\varepsilon = \frac{a-b}{a+b}$; a, b are half-axis of the ellipse, $a = 1 + \varepsilon$, $b = 1 - \varepsilon$; $(\tilde{\rho}, \lambda)$ – polar coordinates of the points in the plane ζ.

Research results. Provisionally separating the plate from its reinforcement, and replacing the influence of one body on another with unidentified contact forces T_ρ, $S_{\rho\lambda}$ applied to Γ_1 we get the first substantive problem for orthotropic plate with unreinforced elliptical hole and elastic beam.

Stressed state in the plate is created by the load applied to infinities and contact forces applied to Γ_1, and in the reinforcement – by contact forces only.

Deformations of Γ_1 contour in orthotropic plate at a given load in the notation [2] are determined by formulas

$$\varepsilon_\lambda = \frac{1}{2E_x h(\alpha^2 + \beta^2)} \Bigg\{ (\beta_1\beta_2 - \nu_1)(\alpha^2 + \beta^2) T_\rho(\lambda) - \frac{1}{\pi} \int_{\alpha_0}^{\beta_0} \left[R_1(\lambda, t) - Q_1(\lambda, t) ctg \frac{\lambda - t}{2} \right] T_\rho(t)\, dt + \\ + \frac{1}{\pi} \int_{\alpha_0}^{\beta_0} \left[Q_1(\lambda, t) + R_1(\lambda, t) ctg \frac{\lambda - t}{2} \right] S_{\rho\lambda}(t)\, dt + \alpha \tilde{\varepsilon}_\lambda^0 + \beta \tilde{V}^0 \Bigg\};$$

$$V = \frac{1}{2E_x h(\alpha^2 + \beta^2)} \Bigg\{ (\beta_1\beta_2 - \nu_1)(\alpha^2 + \beta^2) S_{\rho\lambda}(\lambda) + \frac{1}{\pi} \int_{\alpha_0}^{\beta_0} \left[R_2(\lambda, t) - Q_2(\lambda, t) ctg \frac{\lambda - t}{2} \right] S_{\rho\lambda}(t)\, dt + \\ + \frac{1}{\pi} \int_{\alpha_0}^{\beta_0} \left[Q_2(\lambda, t) + R_2(\lambda, t) ctg \frac{\lambda - t}{2} \right] T_\rho(t)\, dt + \alpha \tilde{V}^0 - \beta \tilde{\varepsilon}_\lambda^0 \Bigg\}, \qquad (2)$$

where

$$\tilde{\varepsilon}_{\lambda}^{0} = p\left[a\sin^2\lambda - b\beta_1\beta_2\cos^2\lambda + b\left(\beta_1+\beta_2\right)\sin^2\lambda\right] + \\ +q\left[a\left(\beta_1+\beta_2\right)\cos^2\lambda - a\sin^2\lambda + b\beta_1\beta_2\cos^2\lambda\right]\beta_1\beta_2; \quad \alpha+i\beta=\omega'\left(\sigma\right); \quad \sigma=e^{i\lambda}; \tag{3}$$

$$\tilde{V}^{0} = p\left[a+b\left(\beta_1+\beta_2+\beta_1\beta_2\right)\right]\sin\lambda\cos\lambda - q\left[a\left(1+\beta_1+\beta_2\right)+b\beta_1\beta_2\right]\beta_1\beta_2\sin\lambda\cos\lambda;$$

E_x, ν_x are Young's modulus and Poisson's ratio of the plate material in the direction of the axis Ox; β_1, β_2 are the roots of the characteristic equation [5]; $\left[\alpha_0, \beta_0\right]$ – image of the reinforced area $\left[\alpha_0^*, \beta_0^*\right]$ with the projection (1); ε_λ, V – relative lengthening of Γ contour and the angle of rotation of its normal.

If the contact forces are known, the ring efforts T_λ can be determined by the formulas given in [2].

Reinforcement beam is simulated by a curved rod, geometrical axis of which does not match with Γ_1. Its stressed state is characterized by longitudinal force N and transverse force Q and bending moment L_b arising in cross-sections and assigned to the beam axis [6]

$$N\left(\lambda\right)\left|\omega'\left(\sigma\right)\right| = bf_1\left(\lambda\right)\cos\lambda + af_2\left(\lambda\right)\sin\lambda; \qquad Q\left(\lambda\right)\left|\omega'\left(\sigma\right)\right| = af_1\left(\lambda\right)\sin\lambda - bf_2\left(\lambda\right)\cos\lambda;$$

$$L_b\left(\lambda\right) = \eta N\left(\lambda\right) + \int_{\alpha_0}^{\lambda}\left[af_1\left(t\right)\sin t - bf_2\left(t\right)\cos t\right]dt. \tag{4}$$

here

$$f_1\left(\lambda\right) + if_2\left(\lambda\right) = i\int_{\alpha_0}^{\lambda}\left[T_\rho\left(t\right) + iS_{\rho\lambda}\left(t\right)\right]\sigma_1\omega'\left(\sigma_1\right)dt; \qquad \sigma_1 = e^{it}. \tag{5}$$

Deformations of reinforcement fiber, which is in contact with the plate, are determined from the ratio [6, 7]

$$\varepsilon_\lambda^{(c)} = \frac{1}{g_4}\left[N + \frac{\rho - r_0}{\rho}\frac{L_b}{R - r_0}\right]; \qquad \frac{d\theta_b}{d\theta} = \frac{1}{g_4}\left[N + \frac{L_b}{R - r_0}\right], \tag{6}$$

where $\varepsilon_\lambda^{(c)}$, θ_b is relative elongation of the fiber and elastic turn angle of the normal to it; $g_4 = E_0F_0$ tensile (compression) reinforcement strength; ρ, R, r_0, are curvature radiuses of the considered, axial, and neutral for pure bending reinforcement fiber respectively; E_0 – Young's modulus of reinforcement material; θ is an angle of normal inclination to the axis Ox; $e^{i\theta} = e^{i\lambda}\,\omega'\left(\sigma\right)/\left|\omega'\left(\sigma\right)\right|$.

The normal stresses that occur in the fiber with a radius of curvature ρ^*, are determined by Hooke's law

$$\sigma^{(c)} = \frac{1}{F_0}\left[N + \frac{\rho^* - r_0}{\rho^*}\frac{L_b}{R - r_0}\right], \tag{7}$$

and tangential tensions in cross-section are determined by Zhuravsky's formula [6].

Considering the contact between the plate and the reinforcement ideal, boundary conditions at their bondage area, taking into account their denotation (5), can be presented as

$$T_\rho \left|\omega'(\sigma)\right|^2 = -af_1'(\lambda)\sin\lambda + bf_2'(\lambda)\cos\lambda; \qquad S_{\rho\lambda}\left|\omega'(\sigma)\right|^2 = -bf_1'(\lambda)\cos\lambda - af_2'(\lambda)\sin\lambda;$$

$$\varepsilon_\lambda = \varepsilon_\lambda^{(c)}; \quad V = \theta_b, \quad \lambda \in [\alpha_0;\ \beta_0]. \tag{8}$$

Substituting (2), (6) to the boundary conditions (8) leads to a system of four singular integral-differential equations with Hilbert kernels to determine the contact efforts T_ρ, $S_{\rho\lambda}$, and functions f_1, f_2. This system should be supplemented with conditions of reinforcement equilibrium

$$f_1(\beta_0) = f_2(\beta_0) = 0; \qquad \int_{\alpha_0}^{\beta_0}\left[a f_1(t)\sin t - bf_2(t)\cos t\right]dt = 0. \tag{9}$$

Assuming in the system (2), (8), (9) that $E_x/E_0 = 0$, we obtain a solution to the problem of partial reinforcement of elliptical aperture contour in orthotropic plate by an absolutely rigid beam [2].

With $E_x/E_0 = 0$, for this system we find the solution of the problem for unreinforced elliptic opening [5], and with $\varepsilon = 0$, $E_0 \neq 0$ – that for partially reinforced circular opening.

Approximate solution of the problem. The exact solution of system (2), (8), (9) cannot be found. For its approximate solution it is necessary to establish the structure of the desired functions at the ends of reinforced area.

Given the first two conditions of equilibrium (9) and formula (5) the following can be written

$$f_1(\alpha_0) = f_1(\beta_0) = 0; \qquad f_2(\alpha_0) = f_2(\beta_0) = 0. \tag{10}$$

Correlations (10) suggest that the functions f_1, f_2 are limited to the area of reinforcement, and are equal to zero at its ends.

Based on the first two boundary conditions (8) it can be established that contact forces should be sought in the class of functions unlimited on the ends of the area $[\alpha_0; \beta_0]$.

Given this, an approximate solution of the problem will be determined by method of mechanical quadrature and collocation [1, 2]. This method was used to study the influence of material orthotropy on stress distribution in the plate and reinforcements.

The results of numerical calculation of forces T_ρ, $S_{\rho\lambda}$, T_λ at the contour Γ of the plate and normal stresses $\sigma_1^{(c)}$, $\sigma_2^{(c)}$ in boundary longitudinal reinforcement fibers with $E_0\big/\sqrt{E_x E_y} = 5$; $h_0/h = 4/3$; $h_0/\eta = 3$; $2\eta/\mathrm{R}^* = 0.1$; $\alpha_0 = -\pi/3$; $\beta_0 = \pi/3$; $p = 0$; $q = 1$ are shown in Figures 2 – 4. Characteristics of orthotropic materials and lines that correspond to these materials on the figures are presented in Table 1.

Figure 3 shows tension $\sigma_1^{(c)}$ that corresponds to fiber which is connected with the plate in the section Γ_1. Figure 4 at the bottom shows the distribution of hoop efforts on the contour of unreinforced opening.

Table № 1.
Characteristics of the researched materials

Plate material	β_1	β_2	ν_x	E_x/E_y	Lines
Isotropic material	1	1	0.300	1	————
Glass-epoxy	2.2712	0.7626	0.250	3	— — — — —
Graphite-epoxy	6.9992	0.7144	0.250	25	— · — · — · —
Epoxy-glass	0.4400	1.3100	0.083	$1/3$	··················
Epoxy-graphite	0.1430	1.4010	0.010	$1/25$	— · · — · · — ·

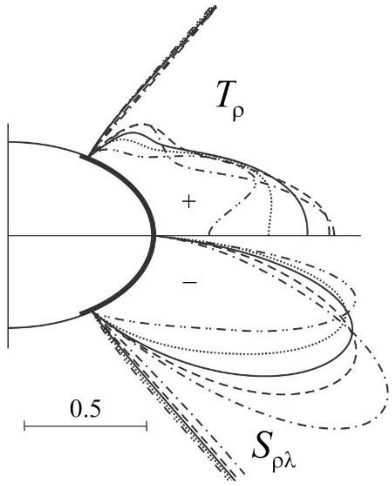

Figure 2. Distribution of contact forces on the area of reinforcement

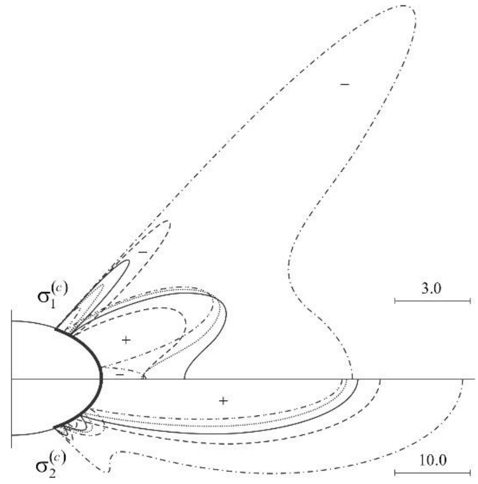

Figure 3. Distribution of stresses in the extreme fiber of reinforcement

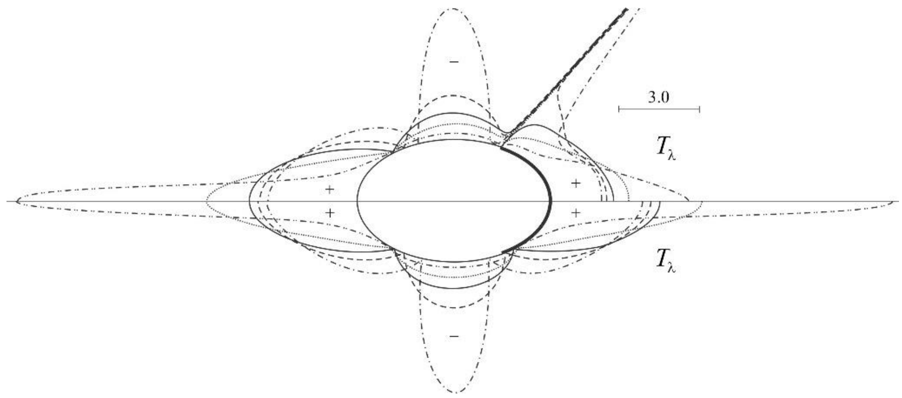

Figure 4. Distribution of hoop efforts on the contour of the hole

Conclusions. As a result of numerical calculations the following has been established:

- reinforcing rib, symmetric by the major axis of the ellipse, at the tension of plate in the direction of the minor axis of the ellipse allows to decrease by half hoop efforts in the areas of maximum concentration at $\lambda = 0$. This is especially true for graphite-epoxide material. Its effects outside areas of reinforcement on the stress state of the plate are practically missed;

- normal stresses in the extreme fiber of reinforcement with the decrementing E_x/E_y are essentially increasing. The maximums of stresses of fiber, which is connected with the plate, are shifted directly to the ends of reinforcement;
- the dependence of the contact forces on the orthotropy of the material of the plate is shown as well as normal stresses in the extreme fibers;
- the efforts at the ends of area of contact in the plate are unbounded, which can be explained by the availability of local plastic areas.

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УДК 539.3

РОЗПОДІЛ НАПРУЖЕНЬ У НЕСКІНЧЕННІЙ ОРТОТРОПНІЙ ПЛАСТИНЦІ З ЧАСТКОВО ПІДСИЛЕНИМ ЕЛІПТИЧНИМ КОНТУРОМ

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Резюме. *Побудовано наближений розв'язок задачі про часткове підсилення контуру еліптичного отвору в нескінченній ортотропній пластинці пружним ребром, поверхня сполучення якого з пластинкою не співпадає з його осьовою поверхнею. Моделюючи підсилювальне ребро криволінійним стрижнем сталого прямокутного поперечного перерізу, побудовано систему інтегральних рівнянь для визначення контактних зусиль між пластинкою та підсиленням і функцій для визначення внутрішніх сил у підсиленні. Числову реалізацію задачі здійснено методом механічних квадратур і колокації.*

Ключові слова: *ортотропна пластинка, підсилювальне ребро, контактні зусилля, сингулярні інтегральні рівняння.*

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