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МЕХАНІКА ТА МАТЕРІАЛОЗНАВСТВО

UDC 539.3

WAVE PROPAGATION IN THE PRE-DEFORMED COMPRESSIBLE ELASTIC LAYER INTERACTING WITH A LAYER OF VISCOUS COMPRESSIBLE LIQUID

Oleksandr Bahno

S.P. Tymoshenko Institute of Mechanics. National Academy of Sciences of Ukraine, Kyiv, Ukraine

***Resume**. Based on three-dimensional equations of linearized elasticity theory for finite deformations of elastic body and three-dimensional linearized Navier-Stokes equations for the liquid medium, the problem of propagation of acoustic waves in preliminarily deformed compressible elastic layer in contact with a layer of viscous compressible liquid has been formulated. A numerical study is conducted, dispersion curves are constructed and dependencies of the phase velocities and attenuation coefficients modes to the thickness of layers of elastic body and a viscous compressible liquid in a wide frequency range are determined. An effect of initial stresses on phase-frequency spectrum of waves in the hydroelastic system is analyzed.*

***Keywords**: compressible elastic layer, layer of viscous compressible liquid, initial stresses, harmonic waves.*

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Problem setting. The development of science and technology brings new increased requirements for research in hydro elasticity and in particular to study wave propagation in elastic bodies in contact with the liquid. There is a strong need for comprehensive consideration of real solid and liquid media properties and on this basis adequate description of different phenomena and mechanical effects that characterize dynamic processes in hydroelastic waveguides.

Analysis of the known research results. The waves propagating along the contact boundary of elastic layer and the layer of liquid are among thoroughly studied generalized basic types of acoustic waves, such as Rayleigh, Stoneley Lyave and Lamb waves. Work reviews and analysis of results obtained within classical elasticity theory and models of ideal compressible liquid are given in [1]. However, considerable practical use of surface waves raises the problem of taking into account real medium properties. Among these factors are the initial tensions and viscosity of the liquid. Tasks examined and results obtained on the basis of the properties of solids and liquids are given in [2, 3].

The purpose of the work. Explore the dispersion spectrum of wave process in a pre-stressed compressible layer – layer of viscous compressible liquid system based on three-dimensional linearized Navier-Stokes equations for the liquid medium and three-dimensional linearized elasticity equations for finite deformation of solids in the most complex theoretical as well as important applied aspect of the case, which covers long-wave and short-wave part of the spectrum.

Formulation of the problem. In this paper, to study wave propagation in a liquid layer – elastic layer system a model is involved that takes into account the initial deformation of solids, together with a model of viscous compressible Newtonian liquid. It uses three-

Corresponding author: Oleksandr Bahno; e-mail: alexbag2010@mail.ru

dimensional equations of linearized elasticity theory at finite deformations of solids and three-dimensional linearized Navier-Stokes equations for the liquid at rest without taking into account thermal effects. The approach chosen applies problem formulation and the method based on the use of representations of general solutions to the equations of motion of an elastic compressible body and a viscous compressible liquid proposed in works [4 – 10].
In the case of homogeneous stress-tension state coefficients in the equations for compressible elastic bodies are constants values that provide a representation of general solutions. For flat case under consideration, the general solution will have the form [4 – 10]

$$u_1 = -\frac{\partial^2 \chi_1}{\partial z_1 \partial z_2};\; u_2 = \frac{\left(\lambda_1^2 a_{11} + s_{11}^0\right)}{\lambda_2^2\left(a_{12} + \mu_{12}\right)}\left[\frac{\partial^2}{\partial z_1^2} + \frac{\lambda_2^2\left(\lambda_1^2 \mu_{12} + s_{22}^0\right)}{\lambda_1^2\left(\lambda_1^2 a_{11} + s_{11}^0\right)}\frac{\partial^2}{\partial z_2^2} - \frac{\rho}{\lambda_1^2\left(\lambda_1^2 a_{11} + s_{11}^0\right)}\frac{\partial^2}{\partial t^2}\right]\chi_1; \tag{1}$$

$$v_1 = \frac{\partial^2 \chi_2}{\partial z_1 \partial t} + \frac{\partial^2 \chi_3}{\partial z_2 \partial t};\; v_2 = \frac{\partial^2 \chi_2}{\partial z_2 \partial t} - \frac{\partial^2 \chi_3}{\partial z_1 \partial t}, \tag{2}$$

where introduced functions χ_i satisfy equation

$$\left[\left(\frac{\partial^2}{\partial z_1^2} + \frac{\lambda_2^2\left(\lambda_1^2 \mu_{12} + s_{22}^0\right)}{\lambda_1^2\left(\lambda_1^2 a_{11} + s_{11}^0\right)}\frac{\partial^2}{\partial z_2^2} - \frac{\rho}{\lambda_1^2\left(\lambda_1^2 a_{11} + s_{11}^0\right)}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2}{\partial z_1^2} + \frac{\lambda_2^2\left(\lambda_2^2 a_{22} + s_{22}^0\right)}{\lambda_1^2\left(\lambda_2^2 \mu_{12} + s_{11}^0\right)}\frac{\partial^2}{\partial z_2^2} - \right.\right.$$

$$\left.\left. - \frac{\rho}{\lambda_1^2\left(\lambda_1^2 \mu_{12} + s_{11}^0\right)}\frac{\partial^2}{\partial t^2}\right) - \frac{\lambda_2^4\left(a_{12} + \mu_{12}\right)^2}{\left(\lambda_1^2 a_{11} + s_{11}^0\right)\left(\lambda_2^2 \mu_{12} + s_{11}^0\right)}\frac{\partial^4}{\partial z_1^2 \partial z_2^2}\right]\chi_1 = 0; \tag{3}$$

$$\left[\left(1 + \frac{4\nu^*}{3a_0^2}\frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial z_1^2} + \frac{\partial^2}{\partial z_2^2}\right) - \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right]\chi_2 = 0; \tag{4}$$

$$\left[\frac{\partial}{\partial t} - \nu^*\left(\frac{\partial^2}{\partial z_1^2} + \frac{\partial^2}{\partial z_2^2}\right)\right]\chi_3 = 0. \tag{5}$$

This problem has the following dynamic

$$P_1\big|_{z_2=0} = Q_1\big|_{z_2=0};\; P_2\big|_{z_2=0} = Q_2\big|_{z_2=0};\; Q_1\big|_{z_2=-h_2} = 0;\; Q_2\big|_{z_2=-h_2} = 0;$$

$$P_1\big|_{z_2=h_1} = 0;\; P_2\big|_{z_2=h_1} = 0 \tag{6}$$

and kinematic

$$v_1\big|_{z_2=0} = \frac{\partial u_1}{\partial t}\Big|_{z_2=0};\; v_2\big|_{z_2=0} = \frac{\partial u_2}{\partial t}\Big|_{z_2=0} \tag{7}$$

boundary conditions. Here are the following notation: u_i – the components of the elastic body travel vector; λ_i – extension of the elastic layer in the directions of coordinate axes; a_{ij} and μ_{ij} – values which are determined from equations of state and depend on the type of elastic potential [11]; $\bar{\sigma}_{ii}^{\ 0}$ – initial stresses ($s_{ii}^{0} = \frac{\lambda_1 \lambda_2 \lambda_3 \bar{\sigma}_{ii}^{\ 0}}{\lambda_{\ i}^{2}}$); ρ – elastic layer matter density; v_i – the components of liquid velocity vector; v^* and μ^* – kinematic and dynamic viscosity of the liquid; ρ_0 and a_0 – the density and speed of sound in a liquid at rest. Q_j and P_j – the components of the stress in a solid and a liquid.

Then parameters, characterizing the propagation of waves, are sought in the class of traveling waves, presented as

$$\chi_j = X_j(z_2) exp[i(kz_1 - \omega t)], \ \ j = \overline{1,3}, \tag{8}$$

where k $(k = \beta + i\gamma)$ – wave number; γ – wave attenuation coefficient; ω – circular frequency.

Note that chosen for this research class of harmonic waves, being the most simple and convenient in theoretical studies, does not limit the generality of the results obtained as a linear wave of arbitrary shape is known to be represented by a set of harmonic components. Then two Sturm-Liouville problems on eigenvalues for equations of travel of an elastic body and liquid are considered. On solving the equations their respective functions are found. After substitution of the solutions into boundary conditions (6) – (7) we get a system of linear homogeneous algebraic equations with reference to integration constants. Based on the conditions of a nontrivial solution existence, and equating the system determinant to zero, we get the dispersion equation

$$det \left\| e_{lm} \left(c, \gamma, a_{ij}, \mu_{ij}, s_{ii}^{0}, \rho_0, a_0, \mu^*, \omega h_1 / c_s, \omega h_2 / c_s \right) \right\| = 0, \ l, m = \overline{1,8}, \tag{9}$$

where c is the phase velocity of waves in hydroelastic system; $c_s (c_s^2 = \mu / \rho)$ – shear wave velocity in the elastic body material; μ – shear modulus; h_1 – thickness layer of the viscous liquid; h_2 – thickness of the elastic layer.

As is known in unlimited compressible elastic body both longitudinal and shear waves exist. In an ideal compressible liquid medium only longitudinal waves spread. Longitudinal as well as and shear waves exist in a viscous compressible liquid. These waves interact in free boundary surfaces, as well as in media contact surfaces, generating a complex wave field in hydroelastic system. Waves, thus created, spread with dispersion. Their phase velocities are in some way dependent on the frequency.

Note that the resulting dispersion equation (9) does not depend on the form of elastic potential. It is the most general and it is possible to obtain a number of partial cases considered in [2, 12 – 14].

Analysis of numerical results. Subsequently the dispersion equation (9) was solved numerically. Herewith the calculations were made for a system of organic glass – water, which is characterized by the following parameters: resilient layer $\rho = 1160$ kg/m3, $\mu = 1{,}86 \cdot 10^9$ Pa; liquid layer $\rho_0 = 1000$ kg/m3, $a_0 = 1459{,}5$ m/s, $\bar{a}_0 = a_0 / c_s = 1{,}1526$, $\bar{\mu}^* = 0{,}001$.

Murnahan form of three-invariant potential was used in numerical realization of a problem for organic glass [11]. With this in view, Murnahan constants for organic glass through which equation values of a_{ij} state and μ_{ij} state, were defined as follows [11, 12]: $a = -3{,}91 \cdot 10^9$ Pa; $b = -7{,}02 \cdot 10^9$ Pa; $c = -1{,}41 \cdot 10^9$ Pa.

The results of calculations are presented in Figures 1 – 8.

For the elastic layer which does not interact with the liquid Fig. 1 shows dependencies of dimensionless values of phase velocities of Lamb waves $\bar{c}$ $(\bar{c} = c/c_s)$ on dimensionless thickness of the elastic layer (frequency) $\bar{h}_2$ $(\bar{h}_2 = \omega h_2/c_s)$ in the absence of initial deformations. Numbers n_a indicate antisymmetric modes and n_s – symmetrical modes accordingly.

Fig. 2 shows the dispersion curves for hydroelastic waveguide showing the dependencies of dimensionless values of phase velocities modes $\bar{c}$ on dimensionless value of viscous liquid thickness $\bar{h}_1$ $(\bar{h}_1 = \omega h_1/c_s)$ for the elastic layer with a thickness equal to $\bar{h}_2 = 10$, and in the absence of initial deformations.

Curves for hydroelastic waveguide showing the dependencies of dimensionless values of mode attenuation coefficients $\bar{\gamma}$ on dimensionless thickness of viscous liquid $\bar{h}_1$ elastic layer with a thickness that equals $\bar{h}_2 = 10$ also in the absence of initial deformations, shown in Fig. 3 – 4.

The nature of the impact of preliminary tension $(\bar{\sigma}_{11}{}^0 = 0{,}004)$ on the phase velocities modes in an elastic layer that interacts with a layer of viscous liquid graphics is illustrated by Fig. 5 – 6, showing the dependencies of the change in the relative phase velocities c_ε $(c_\varepsilon = \dfrac{c_\sigma - c}{c}$; c_σ – phase velocities of modes in hydroelastic system of pre-stressed layer, c – phase velocities of modes in hydroelastic system in the absence of initial deformations) on the thickness of viscous liquid layer for the first 11 modes. These Figures show hydroelastic waveguide dispersion curves, with its elastic layer thickness equal to $\bar{h}_2 = 10$.

The nature of the impact of preliminary tension $(\bar{\sigma}_{11}{}^0 = 0{,}004)$ on the attenuation coefficients of modes in an elastic layer that interacts with a layer of viscous liquid is illustrated on diagrams in Fig. 7 – 9, which shows attenuation coefficient relative value changes dependencies γ_ε $(\gamma_\varepsilon = \dfrac{\gamma_\sigma - \gamma}{\gamma}$, γ_σ – mode attenuation coefficients in hydroelastic system with pre-stressed layer; γ – mode attenuation coefficients in hydroelastic system in the absence of initial deformations) on the viscous liquid thickness for the first 11 modes. These Figures show curves for hydroelastic waveguide with a thick elastic layer, whose thickness is $\bar{h}_2 = 10$.

Research results. From the graphs presented in Fig. 1, it follows that the speed of zero antisymmetric Lamb mode with increasing thickness of the elastic layer (frequency) $\bar{h}_2$ tends to Rayleigh wave velocity $\bar{c}_R$ $(\bar{c}_R = c_R/c_s = 0{,}93356)$ from below, and of zero symmetrical mode speed tends to Rayleigh wave velocity $\bar{c}_R$ $(\bar{c}_R = 0{,}93356)$ from above. Speeds of all higher Lamb modes with increasing thickness of the elastic layer (frequency) tend to shear wave velocity in the material of the elastic body $\bar{c}_s$.

Charts for hydroelastic systems, which are shown in Fig. 2, in the case of thick elastic layer with $\bar{h}_2 = 10$ show that with increasing thickness of the layer of viscous compressible liquid zero antisymmetric mode velocity tends to Stoneley wave velocity $\bar{c}_{st}$ $(\bar{c}_{st} = c_{st}/c_s = 0{,}7691)$, and zero symmetrical mode velocity tends to Rayleigh wave velocity $\bar{c}_R$ $(\bar{c}_R = 0{,}93356)$. By increasing the thickness of the liquid layer the first antisymmetric mode speed tends to wave velocity $\bar{c} = 1{,}1286$, the value of which is less than the speed of sound in a liquid $\bar{a}_0$ $(\bar{a}_0 = 1{,}1526)$. Phase velocities of all other higher modes tend to the speed of sound in a liquid medium $\bar{a}_0$.

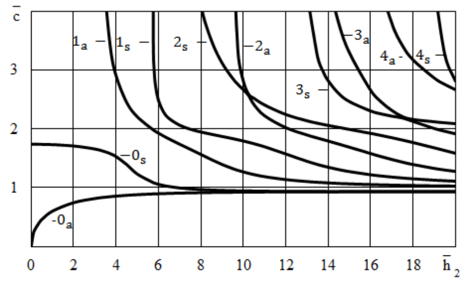

Figure 1. Dependencies of dimensionless phase velocities of Lamb normal waves on the dimensionless thickness of elastic layer in absence of the initial stresses

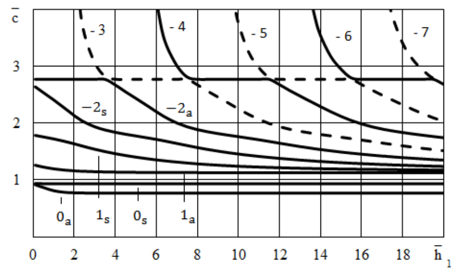

Figure 2. Dependencies of dimensionless phase velocities of modes on the dimensionless thickness of layer of viscous compressible liquid in absence of the initial stresses

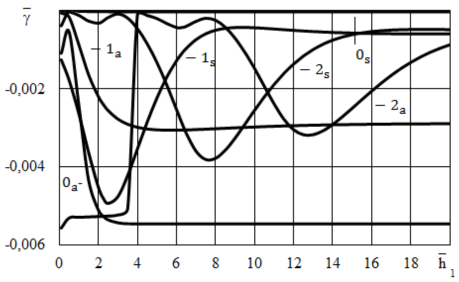

Figure 3. Dependencies of dimensionless attenuation coefficients of modes $0_a, 0_s, 1_a, 1_s, 2_a$ and 2_s on the dimensionless thickness of layer of viscous compressible liquid in absence of the initial stresses

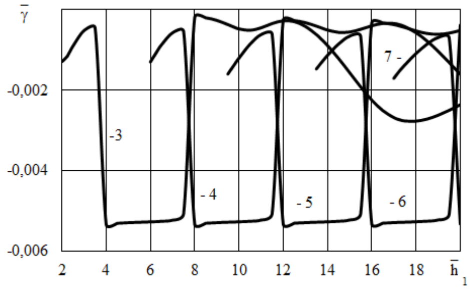

Figure 4. Dependencies of dimensionless attenuation coefficients of modes 3 – 7 on the dimensionless thickness of layer of viscous compressible liquid in absence of the initial stresses

Charts in Fig. 2 show that in hydroelastic waveguide with an elastic layer of a given thickness $\bar{h}_2$ with increasing thickness of the liquid layer $\bar{h}_1$ higher modes velocities tend to the speed of sound in the liquid, which for the considered hydroelastic systems with selected mechanical parameters is greater than shear wave velocity in solid material ($\bar{a}_0 > \bar{c}_s$).

From the graphs presented in Fig. 3 – 4, it follows, that liquid layers of a certain thickness and certain frequencies, for which mode attenuation coefficients take minimum as well as maximum value, exist for all modes. However, for modes 3 – 7 generated by a liquid medium, there are not only certain frequencies, but also the frequency range in which the modes spread with both the smallest and the biggest fading.

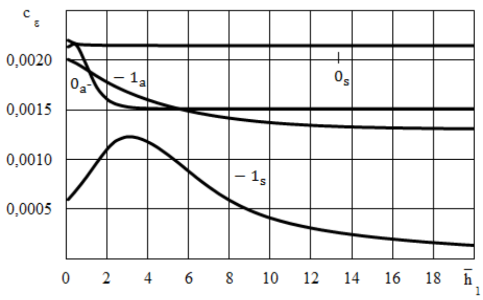

Figure 5. Dependencies of relative changes of phase velocities of modes $0_a, 0_s, 1_a$ and 1_s on the dimensionless thickness of layer of viscous compressible liquid in presence of the initial stretching

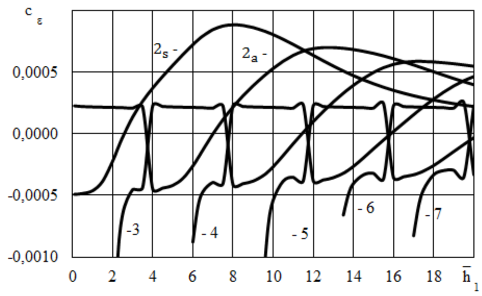

Figure 6. Dependencies of relative changes of phase velocities of modes $2_a, 2_s$, 3 – 7 on the dimensionless thickness of layer of viscous compressible liquid in presence of the initial stretching

From the charts shown in Fig. 5 – 6, it follows that the initial tension of elastic layer causes an increase in phase velocities of zero and first antisymmetric and symmetric modes. Speeds of all higher modes 3 – 7, generated by a layer of liquid in the vicinity of the frequencies of their origin have less velocities of relevant modes in a layer without initial stresses. The impact of the initial tension on the phase velocities of all modes with increasing thickness of the liquid is reduced. It is easy to see that starting with the second mode and onwards on all subsequent there are certain liquid layer thickness and frequencies at which the pre-deformation does not affect their phase velocity. This qualitatively new pattern, which is absent in the case of wave propagation in unbounded and semibounded bodies, was first discovered for the elastic layer that does not interact with the liquid and is presented in work [12]. In the case of thick elastic layer considered here every mode 3 – 7, generated by liquid, has three such frequencies.

We also note that from the charts in Fig. 7 and 8 imply the existence for all modes except 0_s, viscous liquid layers of a certain thickness and certain frequencies at which the pre-deformation does not affect attenuation coefficients of these modes.

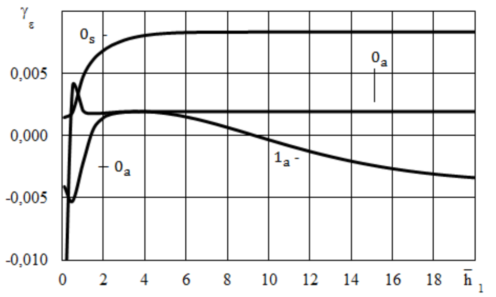

Figure 7. Dependencies of relative changes of attenuation coefficients of modes $0_a, 0_s$ and 1_a on the dimensionless thickness of layer of viscous compressible liquid in presence of the initial stretching

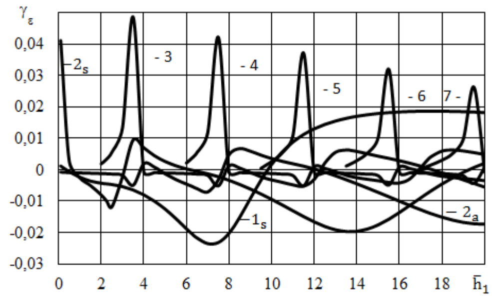

Figure 8. Dependencies of relative changes of attenuation coefficients of modes $1_s, 2_a, 2_s$ and 3 – 7 on the dimensionless thickness of layer of viscous compressible liquid in presence of the initial stretching

Note that the chosen approach, results obtained and identified patterns of mode dispersion spectrum allow for wave processes to set limits of using the models based on different versions of small initial deformations theory as well as perfect liquid model. The results can also be used in ultrasonic non-destructive method of determining the stresses in the surface layers of materials [15] as well as in areas such as seismology, seismic prospecting etc. [11]

Conclusions. Within the framework of the three-dimensional equations of the linearized elasticity theory of finite deformations for the elastic body and three-dimensional linearized Navier-Stokes equations for a viscous liquid of the problem of propagation of acoustic waves in a pre-deformed compressible elastic layer, that interacts with a layer of viscous compressible liquid, was presented. The influence of the initial deformation, the thicknesses of the layers of the elastic body and liquid on the phase velocities and the attenuation coefficients of modes were analyzed. The dispersion curves for the modes in a wide range of frequencies were given. For hydroelastic system it was shown, that with increast of the thickness layer of viscous liquid the velocity of zero antisymmetric mode tends to the Stoneley wave velocity and velocity of zero symmetric mode tends to the Rayleigh wave velocity. By increasing of the thickness of the liquid layer, the velocity of the first antisymmetric mode tends to the wave velocity, the value of which is less than the velocity of sound in the liquid. The phase velocities of all other higher modes tends to the velocity of sound in the liquid. It was determined that the initial tension of the elastic layer leads to the increasing the phase velocities of zero and first antisymmetric and symmetric modes. The velocities of all higher modes which were generated by a layer of liquid in the vicinity of the frequency of their origin are less than relevant velocities in a layer without initial stresses. The effect of the initial tension on the phase velocities of all modes decreases with the increase of layer thickness of the liquid. It was determined that for all the modes, beginning with the second, there exist thicknesses the liquid layer and the certain frequencies, at which the initial tension of the elastic layer has no effect on their phase velocities and attenuation coefficients. It was shown that in the case of thick elastic layer every mode that was generated by the liquid has three such frequencies. An approach developed and the results obtained allow to establish for the wave processes the limits applicability of the models based on different versions of the theory of small initial deformations, as well the model of an ideal liquid. The results can be well used in the ultrasonic non-destructive method determination of stresses in near-the-surface layers of materials as well as in areas such as seismology, seismic, etc.

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УДК 539.

ПОШИРЕННЯ ХВИЛЬ У ПОПЕРЕДНЬО ДЕФОРМОВАНОМУ СТИСЛИВОМУ ПРУЖНОМУ ШАРІ, ЯКИЙ ВЗАЄМОДІЄ З ШАРОМ В'ЯЗКОЇ СТИСЛИВОЇ РІДИНИ

Олександр Багно

Інститут механіки ім. С.П. Тимошенка НАН України, Київ, Україна

Резюме. *На основі тривимірних рівнянь лінеаризованої теорії пружності скінченних деформацій для пружного тіла та тривимірних лінеаризованих рівнянь Нав'є-Стокса для рідкого середовища дано постановку задачі про поширення акустичних хвиль у попередньо деформованому стисливому пружному шарі, що контактує з шаром в'язкої стисливої рідини. Проведено чисельне дослідження, побудовано дисперсійні криві, встановлено залежності фазових швидкостей та коефіцієнтів згасання мод від товщини шарів пружного тіла і в'язкої стисливої рідини у широкому діапазоні частот. Проаналізовано вплив початкових напружень на частотно-фазовий спектр хвиль у гідропружній системі.*

Ключові слова: *пружний стисливий шар, шар в'язкої стисливої рідини, початкові напруження, гармонічні хвилі.*

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