

## Experiment E6

### EXAMINATION OF OHM'S LAW FOR ALTERNATING CURRENT

**Objective:** to examine Ohm's law experimentally; to master methods of AC measurement.

#### 1 EQUIPMENT

- 1) Inductor (coil);
- 2) capacitor;
- 3) AC ammeter and voltmeter;
- 4) two switches;
- 5) rheostat;
- 6) AC and DC sources.

#### 2 THEORY

By definition, current is a flow of electric charge through the cross-section of a conductor. If this flow is constant in time, that is equal amounts of charge pass through the cross-section in equal time intervals, the current is known as direct current (DC). Otherwise the current is alternating (AC). Nowadays, in the power grid the alternating currents are used to provide the power to operate electrical appliances. A sinusoidal alternating current  $I$  is described by formula

$$I = I_{\max} \sin(\omega t), \quad (2.1)$$

where  $I_{\max}$  is a maximum value of current,  $\omega$  is an angular frequency and  $t$  is time.

A circuit consisting of a resistor, an inductor, and a capacitor connected in series, as shown in Figure 2.1, is known as RLC circuit. Let us assume that the resistance of the resistor represents all of the resistance in the circuit.

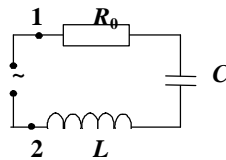


Figure 2.1

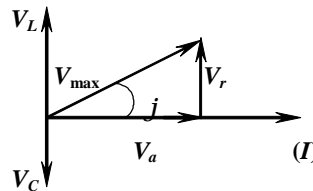


Figure 2.2

An instant value of voltage  $V$  across points 1 and 2 is a sum of voltage drops on the resistor  $R_0$ , the capacitor  $C$  and the inductor  $L$ , which also vary in time, but has some phase shift. Maximum value (amplitude) of voltage  $V_{\max}$  is determined by the vector diagram shown in figure 2.2, where following notations are introduced:

$$V_a = I_{\max} \cdot R, \quad V_r = V_L - V_C,$$

$$V_L = I_{\max} \cdot \omega L, \quad V_C = I_{\max} \cdot \frac{1}{\omega C}.$$

First, we note that because the elements are in series, the current everywhere in the circuit must be the same at any instant. That is, the current at all points in a series AC circuit has the same amplitude and phase. The voltage across each element has a different amplitude and phase. In particular, the voltage across the resistor is in phase with the current, the voltage across the inductor leads the current by  $90^\circ$ , and the voltage across the capacitor lags behind the current by  $90^\circ$ , as it follows from figure 2.2.

In AC circuits that contain inductors and capacitors, it is useful to define the inductive reactance  $X_L$  and the capacitive reactance  $X_C$  as

$$X_L = \omega L,$$

$$X_C = \frac{1}{\omega C}.$$

Time-dependence of voltage is given by formula

$$V = V_{\max} \cdot \sin(\omega t + j), \quad (2.2)$$

where  $j$  is a phase shift between current and voltage,

$$V_{\max} = I_{\max} \cdot \sqrt{R^2 + (X_L - X_C)^2} \quad (2.3)$$

is the maximum value of voltage.

Equation (2.3) is formally identical to Ohm's law and is known as **Ohm's law for alternating current**. Here

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad (2.4)$$

is impedance of RLC-circuit,  $R$  is active resistance,  $(X_L - X_C) = \left(\omega L - \frac{1}{\omega C}\right)$  is reactive resistance (reactance). One can prove the formula (2.4) by determination of  $Z$  in two independent ways: by formula (2.4) and from equation

$$Z = \frac{V_{rms}}{I_{rms}}, \quad (2.5)$$

where  $I_{rms}$  and  $V_{rms}$  are so called *rms* values of current and voltage. The notation *rms* stands for *root-mean-square*, which in this case means the square root of the mean (average) value of the square of the current or the voltage. **AC ammeters and voltmeters are designed to read rms values.**

The *rms* current and *rms* voltage in an AC circuit in which the voltages and current vary sinusoidally are given by the expressions

$$I_{rms} = \frac{I_{\max}}{\sqrt{2}} = 0.707I_{\max}, \quad (2.6)$$

$$V_{rms} = \frac{V_{\max}}{\sqrt{2}} = 0.707V_{\max}. \quad (2.7)$$

where  $I_{\max}$  and  $V_{\max}$  are the maximum values.

The average power delivered by the source in an RLC circuit is

$$P_{av} = I_{rms} V_{rms} \cos j. \quad (2.8)$$

An equivalent expression for the average power is

$$P_{av} = I_{rms}^2 R. \quad (2.9)$$

The average power delivered by the source results in increasing internal energy in the resistor. No power loss occurs in an ideal inductor or capacitor.

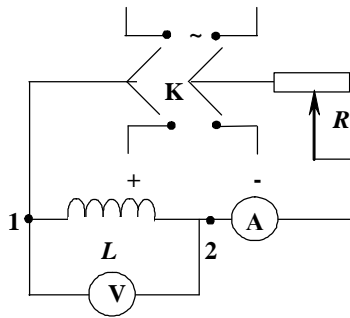


Figure 2.3

A diagram of the circuit designed for determination of impedance  $Z$ , inductance  $L$ , and capacitance  $C$  is given in Fig. 2.3. Here  $K$  denotes a switch used to connect a AC/DC source to the **RLC** circuit, rheostat, ammeter, inductance and voltmeter (connected in parallel to the inductance) are shown.

### 3 PROCEDURE AND ANALYSIS

3.1 Assemble electric circuit according to diagram shown in Fig. 3.1. In the first measurement, inductance  $L$  alone is connected to terminals 1

and 2. First, direct current source is used and DC voltage  $V_0$  and current  $I_0$  are measured. According to Ohm's law, resistance  $R_0$  of the coil can be obtained as

$$R_0 = \frac{V_0}{I_0}.$$

3.2 Using the AC source, measure *rms*-voltage  $V_{rms}$  and *rms*-current  $I_{rms}$ . Calculate the total resistance of the inductor

$$R_1 = \frac{V_{rms}}{I_{rms}},$$

which takes into account both ohmic resistance  $R_0$  of wire and inductance of the coil.

3.3 Calculate inductivity of the coil by formula

$$L = \frac{\sqrt{R_1^2 - R_0^2}}{w};$$

here  $w = 2\pi n$ , where  $n$  is AC frequency (in Europe  $n=50$  Hz).

3.4 Instead the inductor  $L$ , capacitor  $C$  is connected up. Using AC source, *rms*-voltage  $V_{rms}$  and *rms*-current  $I_{rms}$  are measured and capacitor's reactive resistance is calculated by formula

$$R_2 = \frac{V_{rms}}{I_{rms}}.$$

Capacitance  $C$  is calculated as

$$C = \frac{1}{w R_2}.$$

3.5 Instead the capacitor  $C$ , series connection of inductor  $L$  and capacitor  $C$  is connected up.  $V_{rms}$  and  $I_{rms}$  are measured as before and impedance

$$Z_1 = \frac{V_{rms}}{I_{rms}}$$

of the **LC**-segment is calculated.

3.6 Substituting obtained values of  $R_0$ ,  $L$ ,  $C$  into formula

$$Z_2 = \sqrt{R_0^2 + \left( wL - \frac{1}{wC} \right)^2}$$

theoretical estimation of impedance  $Z_2$  is obtained. This estimation is to be compared with experimentally obtained  $Z_1$  value.

3.7 Experimental error for  $Z_1$  is calculated as

$$\frac{\Delta Z_1}{Z_1} = \frac{\Delta V_{rms}}{V_{rms}} + \frac{\Delta I_{rms}}{I_{rms}},$$

where  $DV_{rms}$  and  $DI_{rms}$  are obtained using voltmeter's and ammeter's grades of accuracy, respectively.

3.8 Fill the table 3.1 with results of measurements and calculations.

**Table 3.1**

$V_0$	$I_0$	$R_0$	$V_{rms}^L$	$I_{rms}^L$	$R_1$	$L$	$V_{rms}^C$	$I_{rms}^C$	$R_2$	$C$

$V_{rms}$	$I_{rms}$	$Z_1$	Voltmeter's grade of accuracy	Ammeter's grade of accuracy	$DZ_1$	$Z_2$